A introduction to the contamination background study for Moller

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1 Introduction

This document is an introduction of the contamination background study for Moller project. The primary goal is to extract the Moller asymmetry projections considering all the backgrounds (ep-elastic, ep-inelastic, eAl-elastic, eAl-inelastic) [1] and check how the error is affected by the backgrounds. As one can see from Moller MIE figure 15 and figure 20, the asymmetries measured in all the rings are the combinations of all the processes, although the yield contamination of ep-inelastic is small in rings 1 to 3, the asymmetry contributions are significant.

2 Formalism

In practice, the data were collected by quartz binning on radial and azimuthal two dimensions, see figure 17 of Moller MIE. Hence, for each quartz or (r,ϕ) bin, we have 5 components, N_{ee}, $N_{ep-elastic}$, $N_{ep-inelastic}$, $N_{eAl-elastic}$, $N_{eAl-inelastic}$ at yield level as well as their corresponding asymmetries, A_{ee}, $A_{ep-elastic}$, $A_{ep-inelastic}$, $A_{eAl-elastic}$, $A_{eAl-inelastic}$. For each (r,ϕ) bin, we have

$$A^{(r,\phi)}_{measured} = \frac{N_{ee}}{N_{total}} A^{(r,\phi)}_{ee} + \frac{N_{ep-elastic}}{N_{total}} A^{(r,\phi)}_{ep-elastic} + \frac{N_{ep-inelastic}}{N_{total}} A^{(r,\phi)}_{ep-inelastic} + \frac{N_{eAl-elastic}}{N_{total}} A^{(r,\phi)}_{eAl-elastic} + \frac{N_{eAl-inelastic}}{N_{total}} A^{(r,\phi)}_{eAl-inelastic}; \quad (1)$$

where $N_{total} = N_{ee} + N_{ep-elastic} + N_{ep-inelastic} + N_{eAl-elastic} + N_{eAl-inelastic}$. The yield of different components in different (r,ϕ) bins can be obtained from simulation. The asymmetries in different (r,ϕ) bins can be obtained from kinematical evolution from the certain (r_0,ϕ_0) bin which we would like to extract the final asymmetries.

In practice, we have many measurements $A_{measured}$ depending on the number of quartz, and want to extract 5 asymmetries in the concerned (r_0, ϕ_0) bin. Then we can form a χ^2 to extract the asymmetries as well as the error matrix for the asymmetries. In i_{th} (r,ϕ) bin, we have

$$A^i_m = f^i_{ee} A^i_{ee} + f^i_{ep-elastic} A^i_{ep-elastic} + f^i_{ep-inelastic} A^i_{ep-inelastic} + f^i_{eAl-elastic} A^i_{eAl-elastic} + f^i_{eAl-inelastic} A^i_{eAl-inelastic} \quad (2)$$

[1] I am only looking at the ep and eAl background.

where i is the i_{th} quartz, A_m^i is the measured asymmetry containing all the components, A_{ee}^i is a function of the A_{ee} asymmetry we want to extract in the certain (r_0,ϕ_0) bin, already scaled by kinematical evolution [2]. f is the yield ratio in formula 1.

The χ^2 is formed as

$$\chi^2 = \sum \frac{(A_m^i - f_{ee}^i A_{ee}^i - f_{ep-elastic}^i A_{ep-elastic}^i - f_{ep-inelastic}^i A_{ep-inelastic}^i - f_{eAl-elastic}^i A_{eAl-elastic}^i - f_{eAl-inelastic}^i A_{eAl-inelastic}^i)^2}{\sigma^2_{A_m^i}} \quad (3)$$

The first order of differential to the χ^2 yields the functions to calculate the value of asymmetries. The second order of differential yields the inversed error matrix. Our concern is about the error matrix, the element of the inversed error matrix is (using A_{ee} as an example):

$$\frac{1}{2}\frac{\partial\chi^2}{\partial^2 A_{ee}^i} = \sum_i \frac{(f_{ee}^i)^2}{\sigma^2_{A_{ee}^i}} \quad (4)$$

Therefore, the inversed error matrix F is [3]

$$F = \begin{pmatrix} \sum_i \frac{(f_{ee}^i)^2}{\sigma^2_{A_m^i}} & \sum_i \frac{f_{ee}^i f_{ep-elastic}^i}{\sigma^2_{A_m^i}} & \ldots \\ \sum_i \frac{f_{ee}^i f_{ep-elastic}^i}{\sigma^2_{A_m^i}} & \sum_i \frac{(f_{ep-elastic}^i)^2}{\sigma^2_{A_m^i}} & \ldots \\ \ldots & & \end{pmatrix} \quad (5)$$

The F^{-1} gives matrix of the errors for all the components as well as their correlations in the (r_0, ϕ_0) bin. The diagonal terms are the final σ^2 of all the asymmetries.

3 Extraction of the Moller asymmetry

There are two scenarios to extract the Moller asymmetry. The first one is to do an "overall analysis" using the data from all the quartz detectors. Since we assume that the asymmetries between different (r,ϕ) bins can be connected by kinematic evolution. Thus, we have 18 independent measurements (A_m) to extract all the asymmetries in a concerned (r_0,ϕ_0) bin. We don't need to worry about the contaminations in this scenario, all the asymmetries as well as their uncertainties and correlations are extracted simultaneously.

The second scenario is that we only look at the data in a certain (r_0,ϕ_0) bin. The Moller asymmetry can be obtained by

$$A_{ee} = \frac{N_{tot}}{N_{ee}} A_m - \frac{N_{ep\ elastic}}{N_{ee}} A_{ep\ elastic} - \frac{N_{ep\ inelastic}}{N_{ee}} A_{ep\ inelastic} - \ldots \quad (6)$$

[2]This can be realized in the current generator.

[3]Here, f^i already includes the kinematical evolution factors.

The corrections due to contaminations should be performed. The uncertainties of asymmetries on the right side of the equation 6 contribute to the final uncertainty of the Moller asymmetry. The final statistical uncertainty for the Moller asymmetry is

$$\frac{N_{tot}}{N_{ee}}\sigma_{A_m}, \tag{7}$$

the systematic uncertainty due to ep elastic process is from the uncertainty of $A_{ep\ elastic}$, i.e.

$$\frac{N_{ep\ elastic}}{N_{ee}}\sigma_{A_{ep\ elastic}}, \tag{8}$$

the systematic uncertainty due to ep in-elastic process is from the uncertainty of $A_{ep\ inelastic}$, i.e.

$$\frac{N_{ep\ inelastic}}{N_{ee}}\sigma_{A_{ep\ inelastic}}. \tag{9}$$

The uncertainties, $\sigma_{A_{ep\ elastic}}$, $\sigma_{A_{ep\ inelastic}}$ etc., are from the "overall analysis". This will take advantage of the statistics of the experiment, hence minimize the systematic uncertainties.

3.1 First scenario

For the current study, only Moller, ep-elastic, ep-inelastic processes are included. The following tables summarized the projections from the "overall analysis" by changing the concerned (r_0,ϕ_0) bins. Table 1, 2, 3 show the different results [4] by selecting the open, transition, closed sectors in ring 5 as the concerned bin. By comparing the numbers among the tables, the projections in different bins are different, but the $\frac{\delta_A}{A}$ are the same. These results are extracted using the same set of data, the "evolution factors" affect the error projections in different bins, but the $\frac{\delta_A}{A}$, which should be fixed [5], presents the analyzing power of the data taking.

Physics processes	Expected asymmetry (ppb)	Projections (ppb)	$\frac{\delta_A}{A}$	$\frac{\delta_{Q_W^{e(p)}}}{Q_W^{e(p)}}$
Moller	35.2	0.624	1.77%	1.77%
ep-elastic	16.02	0.653	4.08%	4.08%
ep-inelastic	405.46	12.14	2.99%	No modeling

Table 1: The "overall analysis" of the projections, the concerned (r_0,ϕ_0) bin is selected as ring 5, open sector.

[4]Beam time: 235 + 95 + 14 days. Pol=80%.

[5]It is a fixed number by selecting any (r,ϕ) bin as the concerned bin in the "overall analysis".

Physics processes	Expected asymmetry (ppb)	Projections (ppb)	$\frac{\delta_A}{A}$	$\frac{\delta_{Q_W^{e(p)}}}{Q_W^{e(p)}}$
Moller	33.96	0.602	1.77%	1.77 %
ep-elastic	18.49	0.753	4.08%	4.08%
ep-inelastic	596.21	17.85	2.99%	No modeling

Table 2: The "overall analysis" of the projections, the concerned (r_0,ϕ_0) bin is selected as ring 5, transition sector.

Physics processes	Expected asymmetry (ppb)	Projections (ppb)	$\frac{\delta_A}{A}$	$\frac{\delta_{Q_W^{e(p)}}}{Q_W^{e(p)}}$
Moller	30.55	0.542	1.77%	1.77%
ep-elastic	15.97	0.651	4.08%	4.08 %
ep-inelastic	883.87	26.46	2.99%	No modeling

Table 3: The "overall analysis" of the projections, the concerned (r_0,ϕ_0) bin is selected as ring 5, closed sector.

3.2 Second scenario

Table 4, 5, 6 show the results using the second scenario, in which the contaminations in each quartz are corrected. The statistical uncertainty is calculated from equation 7, the systematic uncertainties are calculated using equations 8, 9.

Physics processes	Projections (ppb)	Normalized to 33 ppb
Moller	0.907	2.7%
sys due to ep-elastic	0.096	0.29%
sys due to ep-inelastic	0.048	0.15%

Table 4: Projections using the second scenario, the data in ring 5, open sector is used.

Physics processes	Projections (ppb)	Normalized to 33 ppb
Moller	0.989	3%
sys due to ep-elastic	0.074	0.22%
sys due to ep-inelastic	0.04	0.12%

Table 5: Projections using the second scenario, the data in ring 5, transition sector is used.

Physics processes	Projections (ppb)	Normalized to 33 ppb
Moller	1.6516	5%
sys due to ep-elastic	0.0529	0.16%
sys due to ep-inelastic	0.0446	0.135%

Table 6: Projections using the second scenario, the data in ring 5, closed sector is used.