Uncertainties in the Theoretical Calculation of the Parity-Violating Asymmetry in Møller Scattering

A.G. Aleksejevs,[1] S.G. Barkanova,[2] J. Erler,[3] Yu.G. Kolomensky,[4] K.S. Kumar,[5] and M.J. Ramsey-Musolf[6]
(For the MOLLER Collaboration)
[1] *Memorial University, Grenfell*
[2] *Acadia University*
[3] *Universidad Nacional Autónoma de México*
[4] *University of California, Berkeley*
[5] *Stony Brook University, SUNY*
[6] *University of Massachusetts, Amherst*
(Dated: September 15, 2016)

In this report, we discuss the currently available theoretical predictions for the parity-violating asymmetry in Møller scattering and their uncertainties. In light of the experimental uncertainty achievable by the MOLLER experiment at Jefferson Lab, a theoretical uncertainty of order 0.5% or better is required. This theoretical uncertainty will be achieved by implementing two-loop calculations of the electroweak asymmetry, including radiative corrections. We discuss the status of the calculations in two schemes: $\overline{\mathrm{MS}}$ and on-shell, and outline the plans towards achieving the ultimate theoretical uncertainty.

I. INTRODUCTION AND MOTIVATION

The MOLLER experiment [1] has been proposed to carry out an ultraprecise measurement of the parity-violating left-right asymmetry in the scattering of longitudinally polarized 11 GeV electrons off electrons in a liquid hydrogen target in Hall A at Jefferson Laboratory (JLab). A successful measurement would provide unprecedented sensitivity to physics beyond the Standard Model, particularly purely leptonic contact interactions beyond the reach of existing high energy colliders, as well as to 100 MeV-scale dark photons that might have small mixing with the Z^0 boson. It would achieve the most precise determination of the weak mixing angle at low energy, comparable in uncertainty to the best ever single measurement at high energy colliders, and best such uncertainty among potential new measurements being proposed or planned in the next decade, either at colliders or at low energy.

The JLab Program Advisory Committee gave MOLLER an A rating, and recommended the allocation of the full beamtime request of 344 PAC days. In September 2014, the Office of Nuclear Physics at the Department of Energy carried out a Science Review of MOLLER. The presentations to the committee and the resulting report and excerpts of comments from panelists can be found here [2]. There were two recommendations in the report, one addressing backgrounds and the other addressing the theory prediction. This report is in response to the recommendation regarding the theory prediction. Specifically, the panel recommended that the collaboration "complete the full two-loop calculations of radiative corrections in order to document the theoretical prediction and submit a report to the Office of Nuclear Physics by September 15, 2016".

In the following sections, we review the current status of the theoretical predictions at one- and two-loop level, describe the recent work on the two-loop calculations, review the current status of the theory uncertainty in the prediction for the MOLLER asymmetry, elaborate on the remaining tasks needed to complete the full 2-loop calculations, and estimate projected improvement in the theory uncertainty.

II. CURRENT STATUS OF THEORETICAL PREDICTIONS

MOLLER will measure the polarized, parity-violating asymmetry [1] A_{PV} in Møller ($e^-e^- \to e^-e^-$) scattering (Fig. 1):

$$-A_{PV} = A_{LR} \equiv \frac{\sigma_{LL} + \sigma_{LR} - \sigma_{RL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR}} \,, \tag{1}$$

[1] Notice that A_{LR} has the opposite sign compared to the asymmetry A_{PV} defined in the MOLLER proposal [1]. In this definition, A_{LR} in Møller scattering is positive.

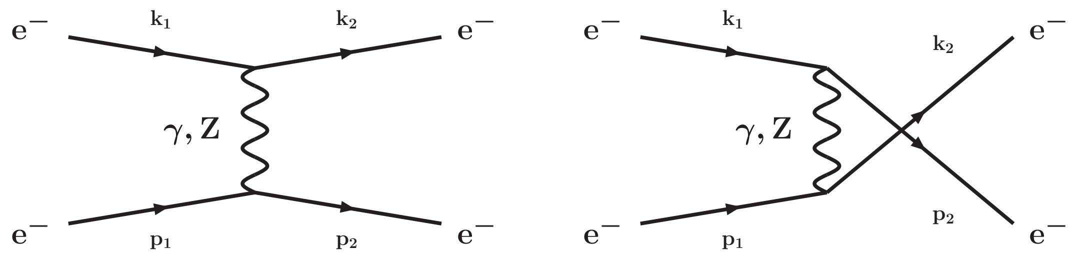

FIG. 1: Born-level diagrams for Møller scattering: t-channel (left) and u-channel (right).

where the cross sections σ_{LL}, σ_{LR}, σ_{RL}, and σ_{RR} are labeled in terms of specific (left, right) helicity states of the two incident electrons. The Born-level asymmetry in the extreme relativistic limit is given by [3]

$$A^0_{LR} \;=\; \mathcal{A}^0(s,y)\left(1-4\sin^2\theta_W\right) = \frac{G_F s}{\sqrt{2}\pi\alpha}\frac{y(1-y)}{1+y^4+(1-y)^4}\left(1-4\sin^2\theta_W\right)\,, \tag{2}$$

where $\mathcal{A}^0(s,y)$ is an experimental acceptance-dependent analyzing power. Lorentz-invariant variables s and $y=-t/s$ are defined by the kinematics of Møller scattering. In terms of laboratory energies of the initial and scattered electrons E and E' and electron mass m, $s=2Em$ is the center-of-mass energy squared, and $y=1-E'/E$. In the kinematics of MOLLER, $E=11$ GeV, E' ranges from 1.7 to 8.5 GeV (corresponding to $y=0.2$-0.8). The average momentum transfer is $\langle Q^2\rangle = 0.0056$ GeV^2.

The Born-level asymmetry is very sensitive to the weak mixing angle $\sin^2\theta_W$, which allows for a precise determination of the fundamental parameter. MOLLER will aim to measure A_{LR} with an experimental uncertainty of about 2%, corresponding to an unprecedented precision of $\sigma(\sin^2\theta_W)_{\rm total}=0.00028$. Any significant deviations from the theoretically-predicted value of A_{LR} would be interpreted as a clean sign of new physics. Therefore, the theoretical prediction has to be robust, ideally with an uncertainty $\sigma(A_{LR})/A_{LR} < 0.5\%$. In particular, given the potential size of two-loop corrections, it is important to tackle the $\mathcal{O}(\alpha^2)$ calculations.

Leading-order $\mathcal{O}(\alpha)$ corrections to the Møller asymmetry have been computed by several groups. These one-loop corrections come from several terms (Fig. 2):

- Vacuum polarization (Fig. 2a) responsible for the "running" of the effective coupling constants. In particular, the $\gamma-Z$ mixing terms which modify the numerator of the asymmetry A_{LR} and produces the largest shift in A_{LR} of 40-60%, depending on the renormalization scheme.

- Vertex corrections and box diagrams (Fig. 2b-f). Electroweak boxes including weak bosons contribute to the numerator of the asymmetry, while the QED (photon) boxes modify the unpolarized cross section (denominator). Overall, this correction to A_{LR} is about 1%

- Radiative effects of soft and hard bremsstrahlung (Fig. 3). Infrared divergences from soft bremsstrahlung are typically canceled against an opposite-sign contributions from the vertex and box diagrams, so the full one-loop calculation needs to include all $\mathcal{O}(\alpha)$ corrections. The hard bremsstrahlung corrections modify the kinematics of Møller scattering, changing both the numerator and the denominator of the asymmetry. For MOLLER, they are expected to be of order 2.5% [4].

Electroweak calculations, including the radiative corrections, are typically computed in two renormalization schemes: $\overline{\rm MS}$ and on-shell (OS). In the OS scheme, the weak mixing angle is defined to all orders in perturbation theory as $\sin^2\theta_W \equiv 1-m_W^2/m_Z^2$, where m_W and m_Z are the physical masses of the $W^\pm$ and Z^0 bosons, respectively[2]. In the $\overline{\rm MS}$ scheme, the weak mixing angle is defined as the ratio of couplings $\sin^2\hat\theta_W = g'^2/(g^2+g'^2)$, where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings. In the $\overline{\rm MS}$ scheme, it is natural to incorporate some of the IR-finite loop corrections, e.g. the vacuum polarization, into the evolution of the weak mixing angle with Q^2, otherwise known as *running* of $\sin^2\theta_W$.

While the OS scheme offers advantages when computing radiative corrections and a simple physical interpretation of the weak mixing angle, the $\overline{\rm MS}$ scheme tends to produce faster convergence of the perturbative expansion. For

[2] One should keep in mind, however, that masses of unstable particles cease to be well-defined starting at the 2-loop level. For a gauge-invariant definition one may choose the real part of the complex pole of the propagator, but this differs from the usually quoted experimental masses of the W and Z bosons.

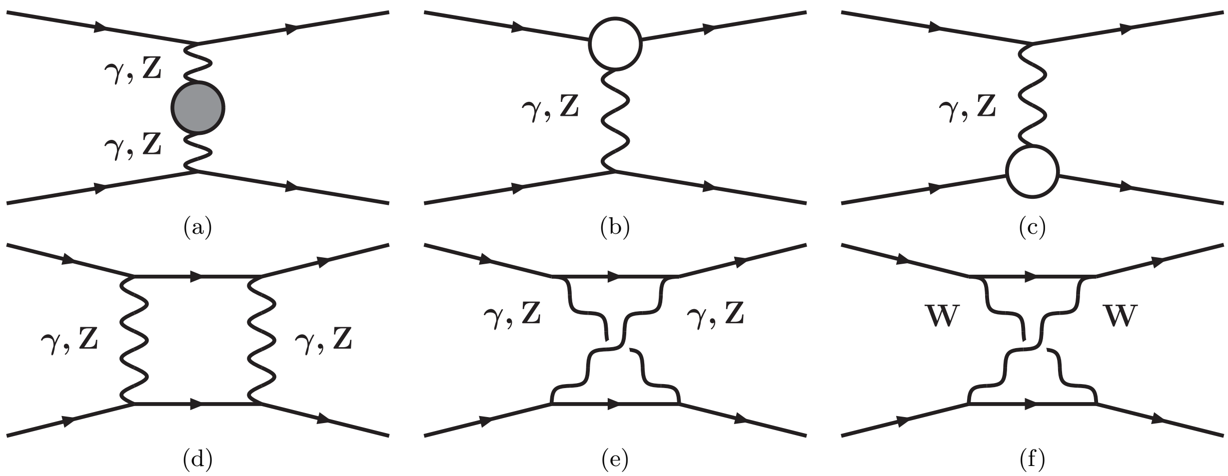

FIG. 2: One-loop t-channel contributions to Møller scattering. Circles represent vacuum polarization (a) and vertex correction contributions (b,c). Adapted from Ref. [4].

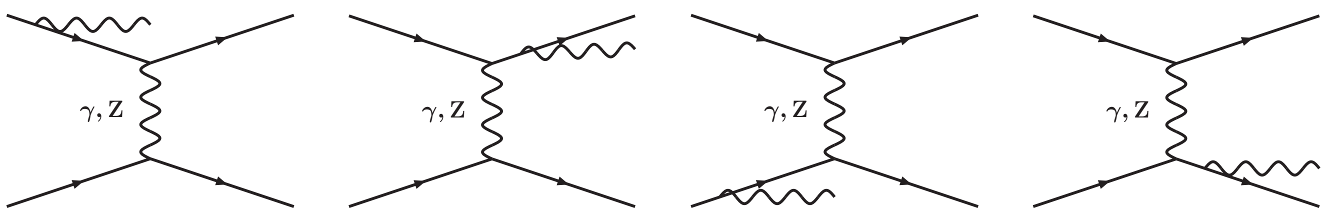

FIG. 3: t-channel bremsstrahlung contributions to process $e^-e^- \to e^-e^-\gamma$. u-channel contributions are obtained by crossing. Adapted from Ref. [4].

instance, $\sin^2\hat{\theta}_W(m_Z)$ is less sensitive than the OS definition to m_t. Since A_{LR} is the physical observable and is independent of the renormalization scheme, comparison between theoretical predictions computed in two different renormalization schemes to the same order in perturbation theory estimates the uncertainty due to the perturbative truncation, *i.e.*, the higher-order terms [3]. Such a comparison between the OS and $\overline{\text{MS}}$ schemes is an important crosscheck of the respective estimates of the independent theoretical uncertainties.

In the next section, we review the current status of the $\mathcal{O}(\alpha)$ calculations of A_{LR} in Møller scattering in the OS and $\overline{\text{MS}}$ schemes.

A. Current Status of the Calculations in $\overline{\text{MS}}$ Scheme

$\mathcal{O}(\alpha)$ corrections to A_{LR} in the $\overline{\text{MS}}$ scheme have been reported in several publications [3, 5, 6], although only the latter attempted a more or less complete treatment including bremsstrahlung. As in the OS scheme, the largest correction to A_{LR} comes from the hadronic vacuum polarization diagrams (Fig. 2a). Ref. [5] systematically relates the hadronic contributions to the $\overline{\text{MS}}$ value of $\sin^2\hat{\theta}_W(0)$ to the vacuum polarization correction of the fine structure constant $\Delta\alpha^{(5)}_{\text{had}}$ using the renormalization group equations (RGE). In addition, a flavor separation analysis [5, 7] of the $e^+e^- \to$ hadrons data needs to be performed using theoretical constraints. The group of J. Erler is currently working on an update of the calculation in Ref. [5], which is expected to reduce the uncertainty (from hadronic loops) to at most 0.5%. This will be achieved using new e^+e^- annihilation data, as well as recent theoretical results regarding hadronic τ decays [8]. Furthermore, augmenting such an analysis with data on ϕ and kaon production may reduce this uncertainty even further. The technique has the advantage of summing large logs associated with light quark masses, and provides a consistent framework for analyzing the associated uncertainties.

For the $\overline{\text{MS}}$ scheme, we can use the results of global electroweak fits [9] to estimate the contributions of the Z

[3] One has to be careful about using a consistent set of input parameters for all calculations.

pole and other observables. They are listed in Table II. There are two estimates of the $\mathcal{O}(\alpha^2)$ uncertainties. One, computed in Ref. [4], accounts for uncertainty in scale at which the weak mixing angle in the $\mathcal{O}(\alpha)$ electroweak box diagrams is evaluated. It is (conservatively) estimated by varying the scale from 0 to m_Z to be of $\mathcal{O}(2\%)$. Ref. [6] argues that even if the large logs are not summed as in Ref. [5], the $\mathcal{O}(\alpha^2)$ correction should not be larger than $[\alpha/\pi \log(m_Z^2/m_e^2)]^2 \approx 7\%$ of Q_w^e. Both of these point to potentially percent-level $\mathcal{O}(\alpha^2)$ effects.

B. Current Status of the Calculations in On-Shell Scheme

A set of one-loop $\mathcal{O}(\alpha)$ corrections to A_{LR} in Møller scattering, applied to the kinematics of the E158 experiment [10], *i.e.* beam energy of ~ 50 GeV, were computed in Ref. [4]. This calculation included bremsstrahlung corrections and contributions from diagrams in Fig. 2b-f, but deliberately omitted finite vacuum polarization corrections (Fig. 2a), which were computed elsewhere [3]. This "hybrid" approach allowed E158 to extract the "effective" value of the low-energy weak mixing angle $\sin^2\theta_W^{\rm eff}$, as well as the value of the $\overline{\rm MS}$ coupling $\sin^2\hat{\theta}_W(M_Z)$. At the time of E158, the theoretical uncertainty was dominated by the estimate of the hadronic vacuum polarization contributions (Fig. 2a), which contributed an uncertainty of 0.3% to $\sin^2\theta_W^{\rm eff}$, or 1.3% to A_{LR}, and an estimate of the higher-order $\mathcal{O}(\alpha^2)$ contributions to the electroweak box diagrams (Fig. 2de), estimated to be 1.9%. Uncertainties due to hard bremsstrahlung radiative corrections are experiment-dependent; E158 conservatively estimated an error of 0.5%.

A complete and self-consistent $\mathcal{O}(\alpha)$ calculation in the OS scheme was performed in Refs. [11, 12]. All graphs in Fig. 2-3 were included. The vacuum polarization correction (Fig. 2a) was computed using a one-loop quark-level formula, but using the values of the effective quark masses from the fit to the running of α_{QED}. The argument that the running of α constrains the hadronic contributions to the $\gamma - Z$ mixing is qualitatively similar to the approach taken in Ref. [5] and relies on the fact that the hadronic vector current, rather than the axial current, enters into both $\Pi_{\gamma\gamma}$ and $\Pi_{\gamma Z}$ vacuum polarization functions. Since these calculations are similar, we will use the uncertainty on the hadronic contributions from Ref. [5].

Consistent with the calculations in the $\overline{\rm MS}$ scheme, the OS calculation [12] uses the physical observables [9] α, G_μ, and m_Z as inputs to the theoretical prediction for A_{LR}. The mass m_W of the W boson is computed in the OS scheme using the standard formulae (e.g. Eq. (10.10) in [9]). Table II summarizes the most dominant contributions to the uncertainty on the Standard Model value of A_{LR} as of this writing. There are two types of uncertainties listed. The "parametric" uncertainty of 0.5% comes from the experimentally measured Standard Model inputs.

We note that the parametric uncertainty listed in Table II is only relevant when the value of A_{LR} is compared to the Standard Model prediction, *e.g.* when constraints or discoveries of new physics effects are extracted from the MOLLER data. The parametric uncertainty is significantly smaller when $\sin^2\theta_W^{\rm eff}$ is computed from the MOLLER results.

Theoretical uncertainties come from the incomplete perturbative expansion (high-order corrections), hadronic loops uncertainties, and from radiative QED effects. Uncertainties due to bremsstrahlung corrections are at the level of 0.1%, but depend on the accuracy of the simulations of experimental acceptance. Uncertainties due to the perturbative truncation (*i.e.*, $\mathcal{O}(\alpha^2)$ terms) were estimated in Ref. [12] by comparing the A_{LR} computed in two schemes: OS and CDR (Constrained Differential Renormalization). The latter is similar to $\overline{\rm MS}$ with a redefined regularization scheme. The authors found the difference in the predicted values of A_{LR} of about 3%, indicating a sizable contribution from $\mathcal{O}(\alpha^2)$ terms.

III. $\mathcal{O}(\alpha^2)$ CALCULATIONS

A. Prospects for $\mathcal{O}(\alpha^2)$ Calculations in $\overline{\rm MS}$ Scheme

J. Erler *et al.* have committed to revisiting the uncertainty due to the hadronic vacuum polarization, and expect that uncertainty to be reduced from its current value of 0.6%. Members of the UMass group (M. Ramsey-Musolf, one post-doc and one PhD student) and A. Freitas (U. Pittsburgh) will carry out a complete computation of the $\mathcal{O}(\alpha^2)$ corrections, incorporating the γ-Z mixing terms addressed above. The work is being initiated in Fall 2016. It will draw on a variety of multi-loop methods currently utilized by the high-energy community and reviewed in Ref. [13]. The goal is to complete the computation in 3 years, subject to funding constraints.

Since the dominant contribution due to hadronic vacuum polarization has already been included in the RGE equations [5], additional $\mathcal{O}(\alpha^2)$ corrections in $\overline{\rm MS}$ scheme are expected to be smaller than in OS scheme. Uncertainties due to perturbative truncation of $< 0.1\%$ should be achievable.

TABLE I: Estimates of the $\mathcal{O}(\alpha^2)$ contributions to A_{LR} in OS scheme. Corrections are expressed relative to A_{LR}^{Born}, *i.e.* in terms of the correction $\delta = \Delta A_{LR}/A_{LR}^{\text{Born}}$.

Source	δ	Reference
$\mathcal{O}(\alpha)$	-69.5%	[11]
Quadratic and reducible corrections	$+5.3\%$	[14]
3-boson boxes	-1.14%	[15]
$\mathcal{O}(\alpha^2)$ QED	$+0.34\%$	
VP and vertex corrections in boxes	-0.39%	[17]
$\mathcal{O}(\alpha^2)$ EW vertex corrections	-0.34%	[18]
double VP	?	in progress

TABLE II: Current status of the parametric and theoretical contributions to the uncertainty on the Standard Model A_{LR} prediction.

Source	Uncertainty
Experimental inputs (global fit)	0.5%
Hadronic loop	0.6%
Bremsstrahlung	$< 0.5\%$
Remaining $\mathcal{O}(\alpha^2)$	$< 1\%$

B. Prospects for $\mathcal{O}(\alpha^2)$ Calculations in On-Shell Scheme

The group of Aleksejevs *et al.* has started a systematic evaluation of the two-loop $\mathcal{O}(\alpha^2)$ contributions to A_{LR} [14–16]. This is a Herculean effort: at $\mathcal{O}(\alpha^2)$, one has to include $\mathcal{O}(100)$ diagrams, even before radiative effects are taken into account. So far, the following contributions have been computed: the "quadratic" and "reducible" terms (*i.e.*, products of the $\mathcal{O}(\alpha)$ corrections), including diagrams with two radiated photons, diagrams with two vacuum polarization "bubbles", graphs with two vertex corrections and with vertex corrections and one vacuum polarization loop, 3-boson boxes, vertex corrections and vacuum polarization in boxes, electroweak vertex corrections at $\mathcal{O}(\alpha^2)$, and $\mathcal{O}(\alpha^2)$ QED corrections to the denominator of the asymmetry. Other diagrams, *e.g.* vacuum polarization corrections with a boson propagator, are still yet to be calculated.

The current estimates of the corrections are listed in Table I. Indeed, we observe $\mathcal{O}(5\%)$ corrections to A_{LR}, dominated by the two-loop vacuum polarization terms, and consistent with the estimates of the $\mathcal{O}(\alpha^2)$ truncation error described in Section II. We note that after the dominant quadratic and reducible corrections, the remaining effects are below 1% level.

While the calculation is not yet complete, we can estimate the ultimate theoretical uncertainty associated with the perturbative truncation at $\mathcal{O}(\alpha^2)$ level. Relative to the $\mathcal{O}(\alpha^2)$ correction, we expect no additional enhancement at $\mathcal{O}(\alpha^3)$ beyond the standard large logs of order $(\alpha/\pi)^3 \log^3(m_Z^2/s) \approx 0.15\%$ of Q_w^e. This is smaller than the uncertainty due to the electroweak parameters, namely m_Z. Overall, we expect a theoretical error of order 0.1-0.2% once the two-loop calculations are complete, and a parametric (Z-pole data) uncertainty of the order of 0.4%.

IV. OUTLOOK

The current uncertainty on the Standard Model value of A_{LR} is estimated to be $\lesssim 1.4\%$ (Table II), including both the theoretical error from the partial two-loop calculation and the uncertainty from the Standard Model parameters. $\mathcal{O}(\alpha^2)$ contributions to the parity-violating asymmetry A_{LR} in Møller scattering are expected to be at most $\lesssim 5\%$. We have identified two independent theoretical groups that have committed to undertaking these calculations in two renormalization schemes: $\overline{\text{MS}}$ and on-shell. It is expected that it may take 2-3 years to complete both sets of calculations. The most dominant terms have already been computed in the OS scheme without RGE and log summation and are of that order; RGE calculation of the hadronic vacuum polarization in $\overline{\text{MS}}$ already includes these terms.

Once the $\mathcal{O}(\alpha^2)$ calculations are complete, an estimate of the residual theoretical uncertainty can be obtained by estimating the truncation error, *i.e.* uncertainty beyond two loops (see Section III). Comparison between the two renormalization schemes, when done properly and with the consistent sets of experimental inputs (*i.e.* α, G_μ, and properly computed gauge boson and fermion masses) provides a useful cross check of the truncation error. We expect that at the end of this process, the truncation error will be small, $< 0.1\%$. The predicted value of A_{LR} will then be dominated by the parametric uncertainty (0.4-0.5%). While this would be the relevant theoretical error when the experimental results are compared to the Standard Model in search for new physics contributions, we note that the parametric uncertainties cancel in some cases, *e.g.* when Møller scattering results are compared to the Z-pole observables. Overall, we expect to achieve our goal of a total uncertainty on the predicted value of A_{LR} of order 0.5%, small compared to the expected experimental uncertainty of 2.4%.

We also point out that the effects of hard bremsstrahlung radiation are not negligible at this level. Since the bremsstrahlung corrections are highly sensitive to the details of experimental acceptance, accurate simulations of the radiative effects, both internal and external, are needed. Therefore, we plan to include radiative correction calculations into the event generators used to simulate the MOLLER apparatus.

The MOLLER Collaboration will closely coordinate the theoretical efforts from the two groups towards the completion of the two-loop calculations. Progress will be reviewed annually in the form of informal discussions, collaboration meeting presentations, and workshops, as necessary.

---

[1] J. Benesch *et al.* [MOLLER Collaboration], arXiv:1411.4088 [nucl-ex].
[2] http://hallaweb.jlab.org/12GeV/Moller/scirev2014/
[3] A. Czarnecki and W.J. Marciano, *Phys. Rev.* **D53**, 1066 (1996).
[4] V. A. Zykunov *et al.*, hep-ph/0507287.
[5] J. Erler and M. J. Ramsey-Musolf, Phys. Rev. D **72**, 073003 (2005), hep-ph/0409169.
[6] F. J. Petriello, Phys. Rev. D **67**, 033006 (2003), hep-ph/0210259.
[7] F. Jegerlehner, Z. Phys. C **32**, 195 (1986).
[8] F. Jegerlehner, EPJ Web Conf. **118**, 01016 (2016), arXiv:1511.04473 [hep-ph].
[9] K.A. Olive *et al.*[Particle Data Group], Chin. Phys. C, **38**, 090001 (2014) and 2015 update; J. Erler and A. Freitas, review "Electroweak model and constraints on new physics" and references therein.
[10] P. L. Anthony *et al.* [SLAC E158 Collaboration], Phys. Rev. Lett. **95**, 081601 (2005), hep-ex/0504049.
[11] A. Aleksejevs *et al.*, Phys. Rev. D **82**, 093013 (2010).
[12] A. Aleksejevs *et al.*, Phys.Part.Nucl. 44 (2013) 161-174
[13] A. Freitas, Prog.Part.Nucl.Phys. **90**, 201 (2016).
[14] A. Aleksejevs *et al.*, Phys. Rev. D **85**, 013007 (2012), arXiv:1110.1750 [hep-ph].
[15] A. Aleksejevs *et al.*, Eur.Phys.J. **C72**, 2249 (2012).
[16] A. Aleksejevs *et al.*, Phys.Atom.Nucl. **76**, 888 (2013), Yad.Fiz. **76**, 942 (2013).
[17] A. G. Aleksejevs *et al.*, Phys. Part. Nucl. Lett. **12**, no. 5, 645 (2015), arXiv:1504.03560 [hep-ph].
[18] A. G. Aleksejevs *et al.*, Phys. Part. Nucl. Lett. **13**, no. 3, 310 (2016), arXiv:1508.07853 [hep-ph].