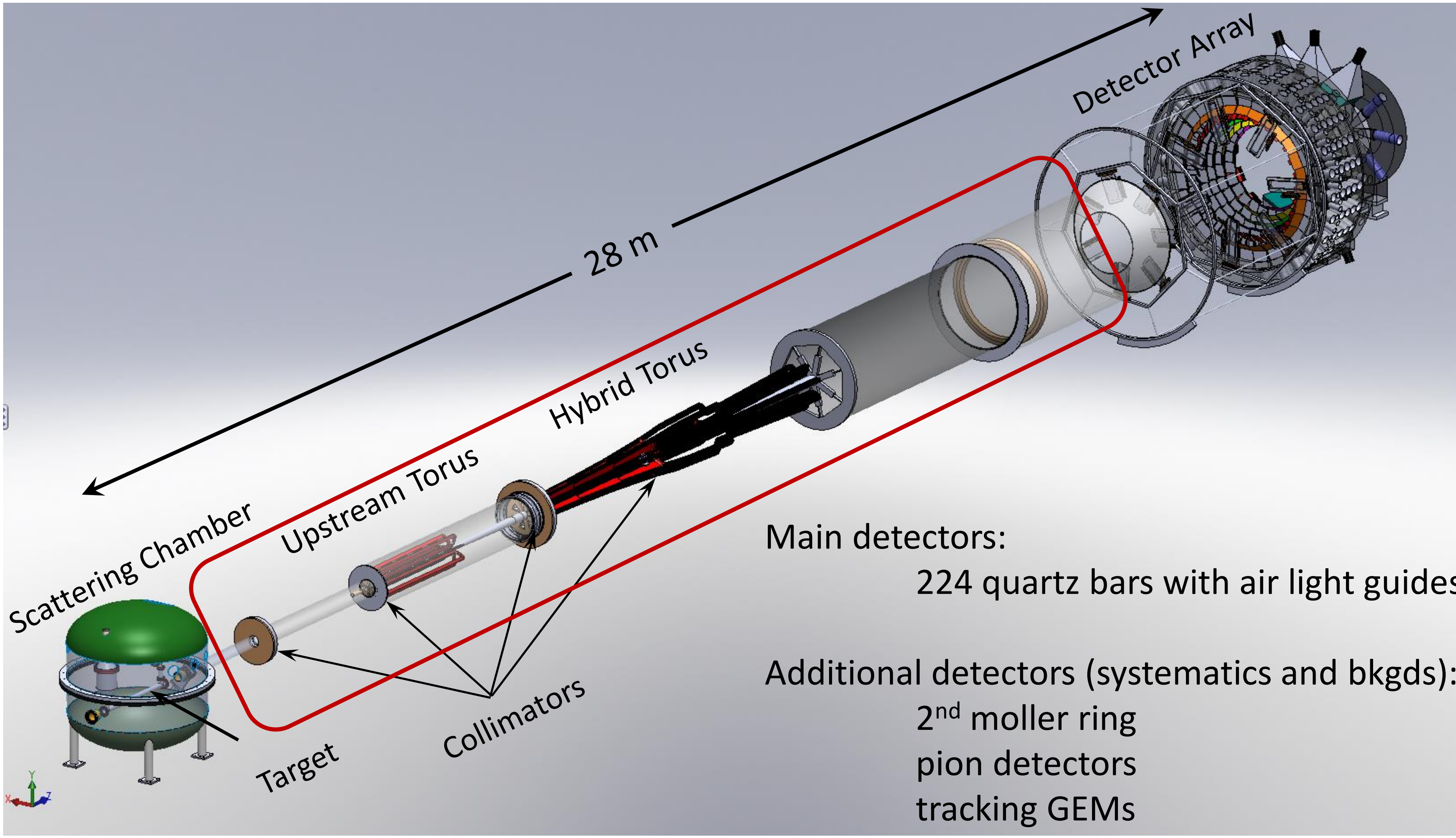
Spectrometer Overview

Juliette Mammei

The Experiment



Main detectors:

224 quartz bars with air light guides

Additional detectors (systematics and bkgds):

2nd moller ring

pion detectors

tracking GEMs

Outline

- Physics overview
- Results of recent studies
- Install/commission /tests
- Alternatives
- Tolerances
- Further work

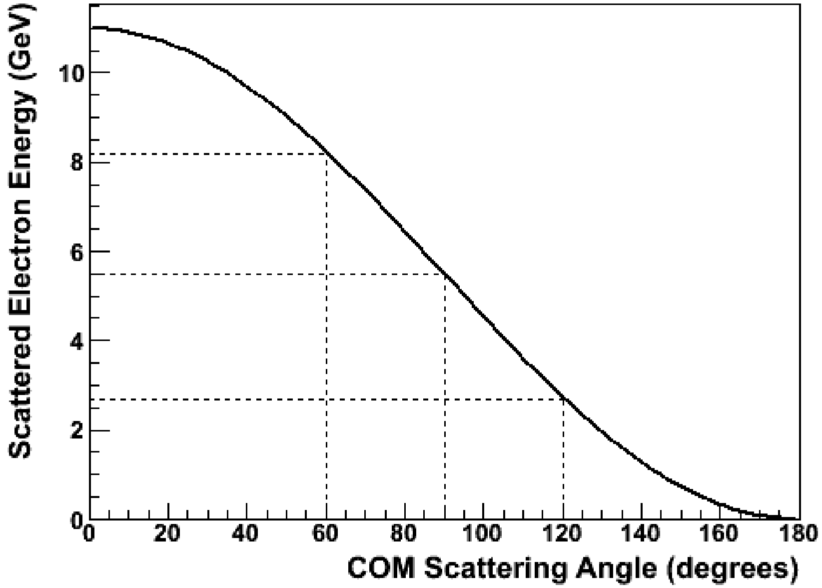
Physics Overview

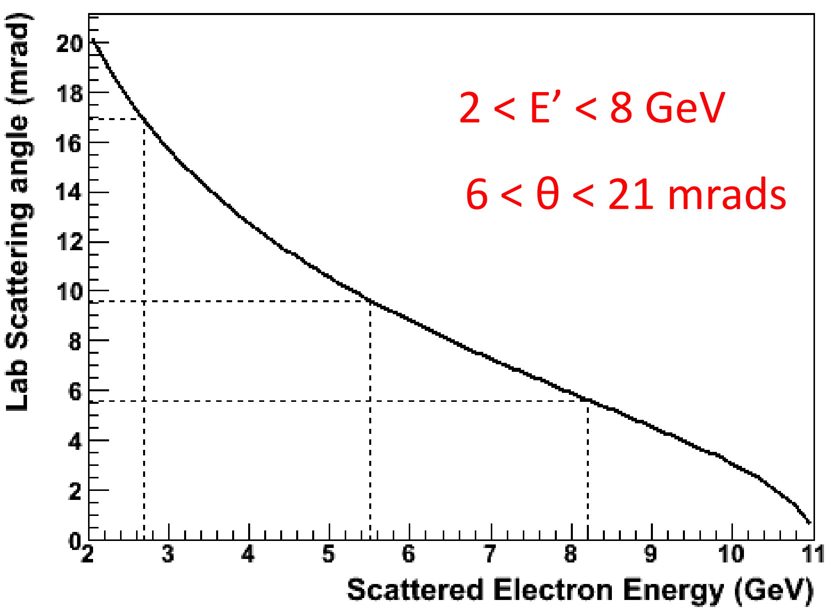
- Full azimuthal acceptance
  - Identical particle scattering
  - Large range of forward-backward scattered electrons
- High energies, forward angles
  - Asymmetry depends on q^2
  - Need to understand acceptance
- Relatively focused mollers
  - Small bkgds
  - Dilution and asymmetry

The signal is an asymmetry

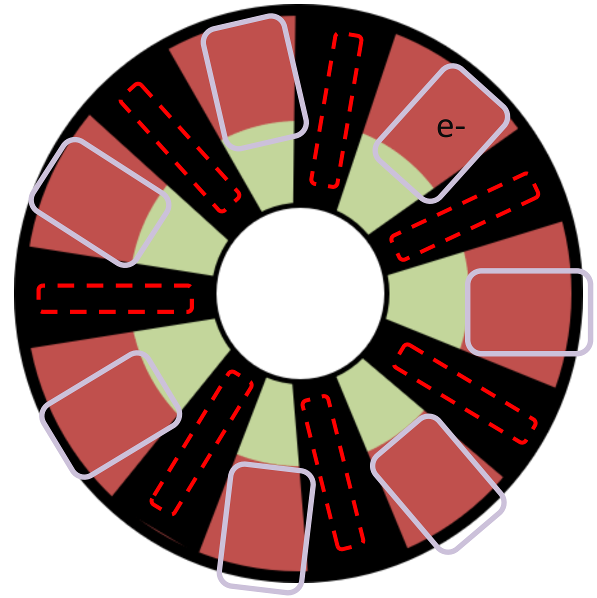
$$A = \frac{Y_+ - Y_-}{Y_+ + Y_-}$$

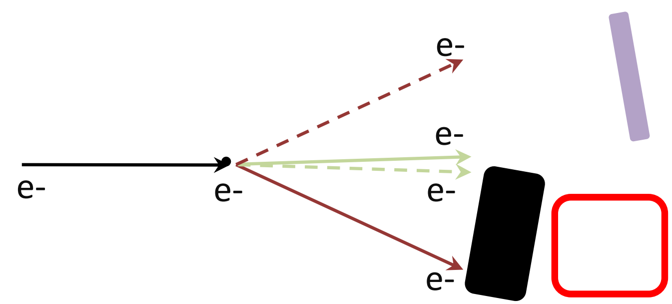
from e- e- scattering





100% Azimuthal Acceptance





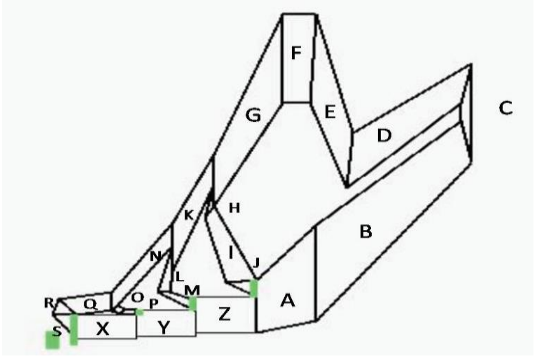
Lab Frame

Any odd number of coils will work

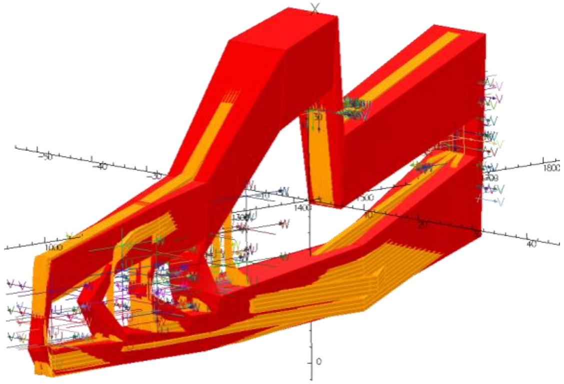
Initial constraints

- Choose (standard) conductor size/layout that minimizes current density
- Try to use "double pancakes"; as flat as possible
  → several out of plane bends
- Minimum bend radius 5x conductor OD
- Fit within radial, angular acceptances (360°/7 and <360°/14 at larger radius)
- Total current in each inner "cylinder" same as proposal model → as close as possible with integer multiples
- Take into account water cooling hole, insulation
- Need to consider epoxy backfill and aluminum plates/ other supports

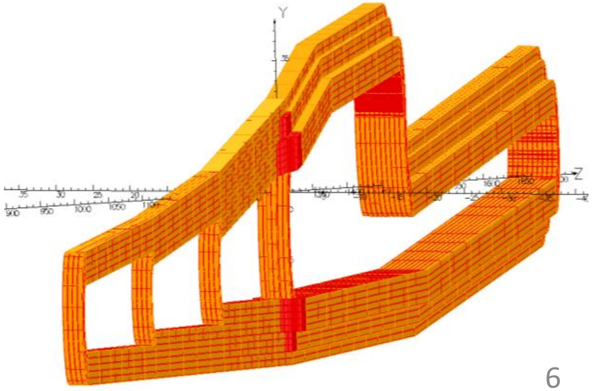
  ➢ Radial extent depends on upstream torus and upstream parts of hybrid!!



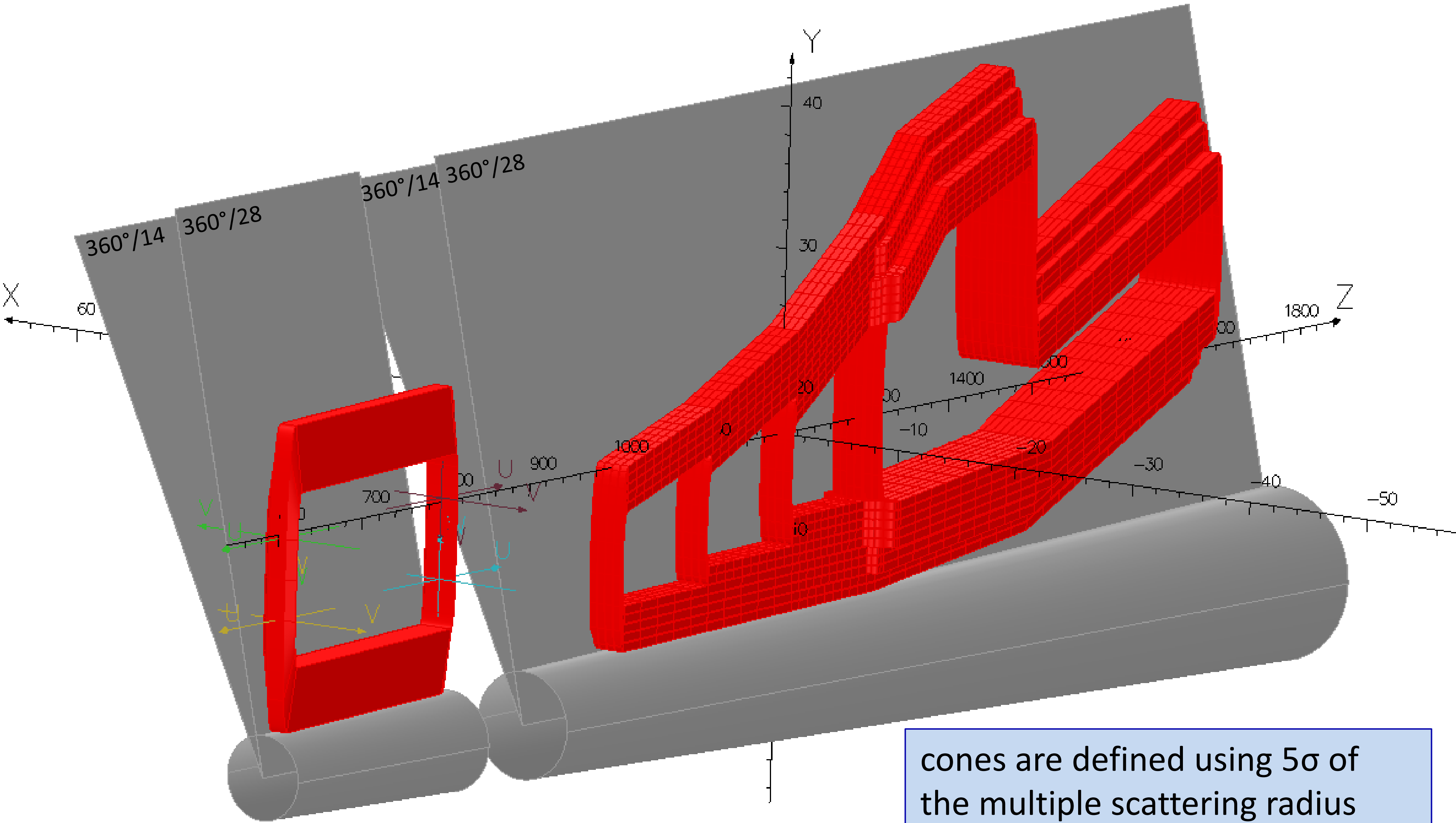
Blocky Actual Layout, with Actual Layout



Actual conductor layout, with blocky version

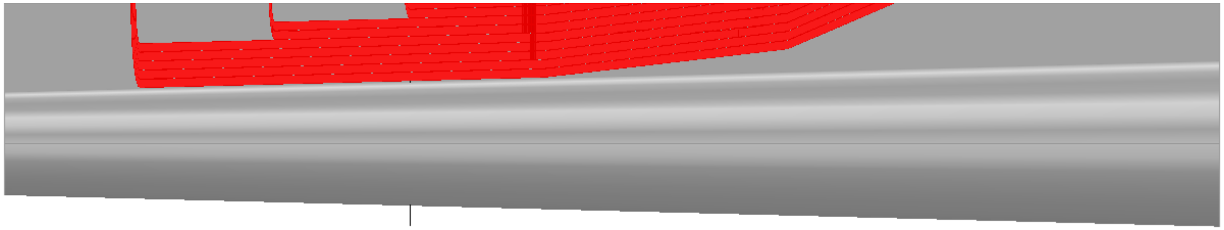


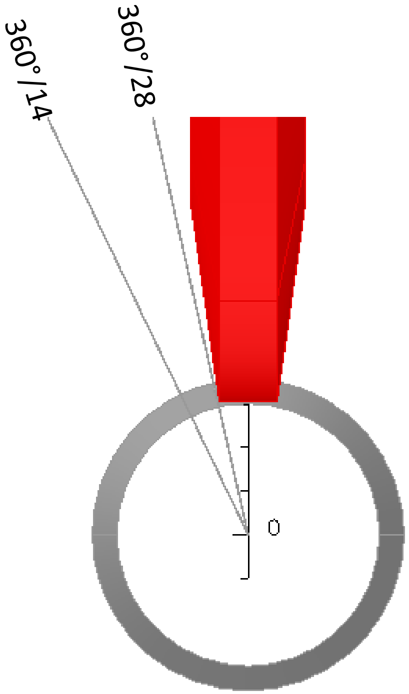
Keep Out Zones

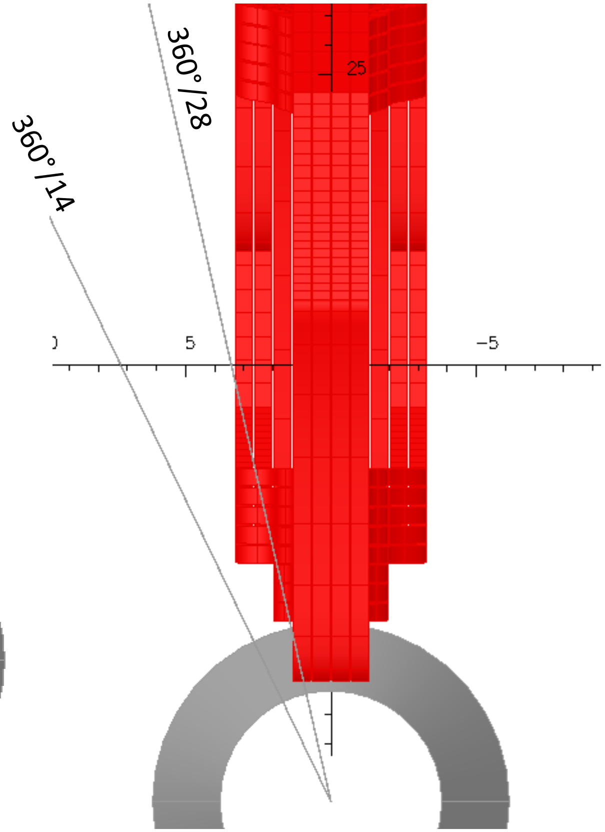


cones are defined using 5σ of the multiple scattering radius

Keep out zones





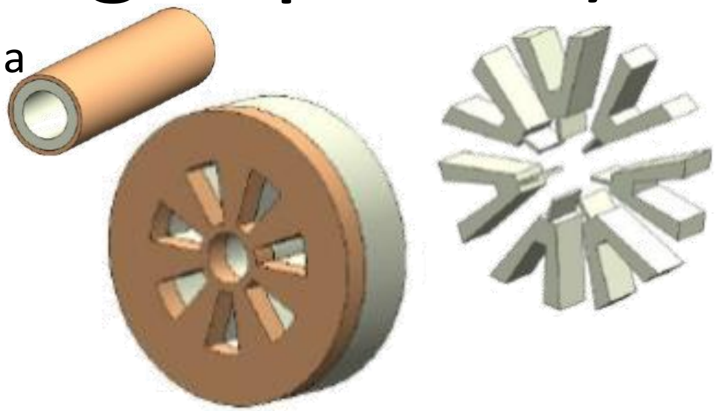


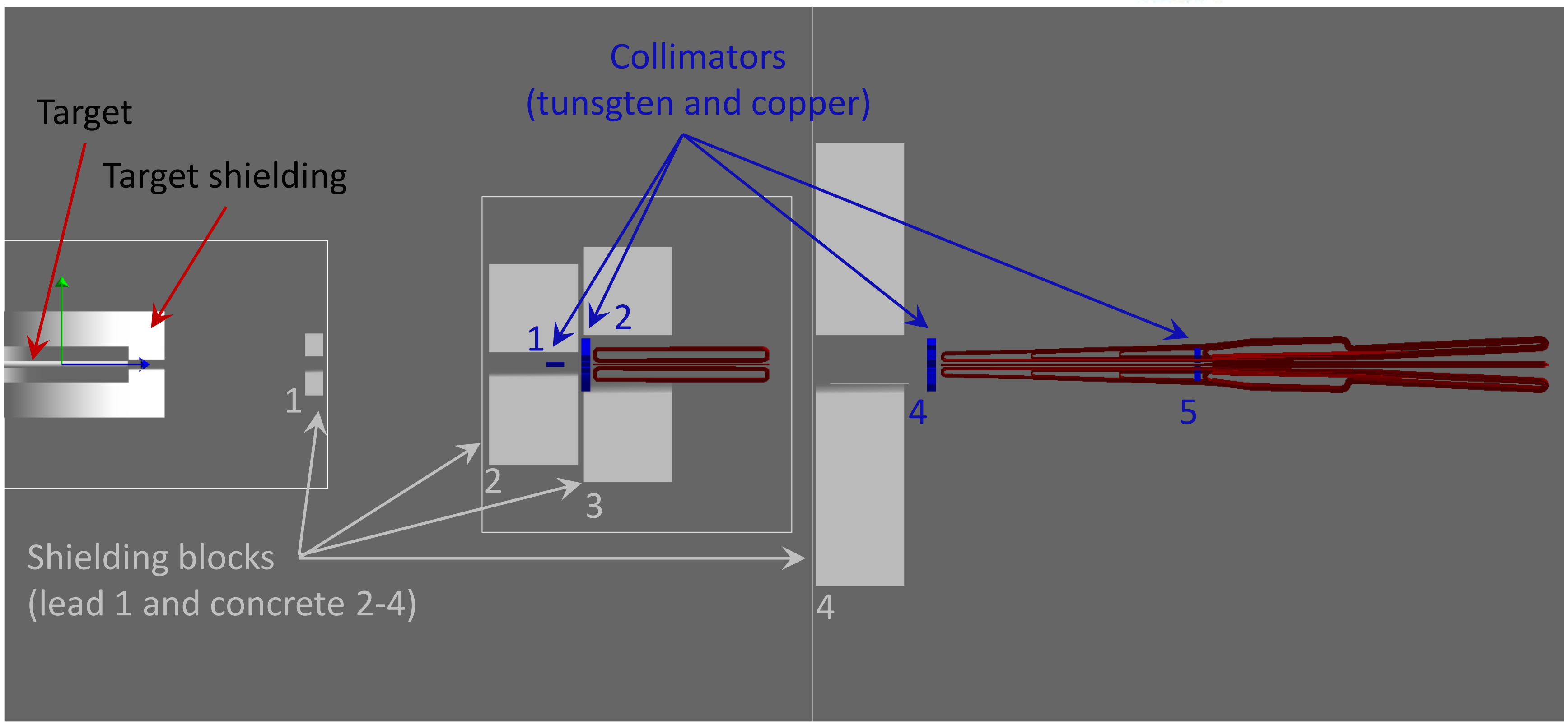
Spectrometer (shielding separate)

Collimators and beam shields are designed to provide a 2-bounce system for photons to detectors

Precision alignment; water-cooling

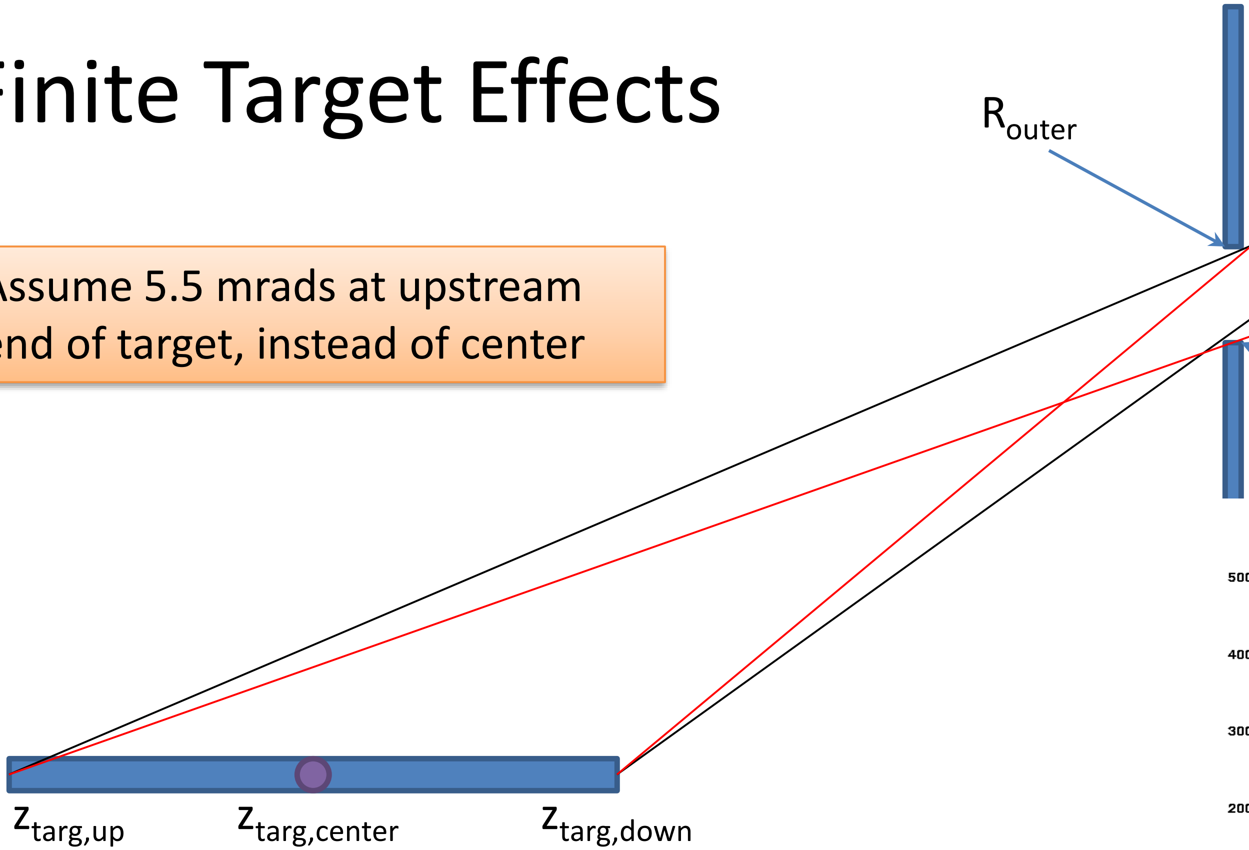
We will require local shielding (mostly due to neutron production) and radiation monitoring



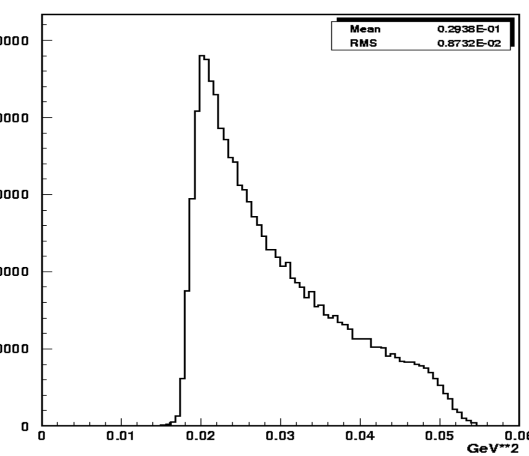


Finite Target Effects

Assume 5.5 mrads at upstream end of target, instead of center



Expected Q^2 distribution



z_{coll} = 590 cm

$z_{targ,up}$ = -75 cm

$z_{targ,center}$ = 0 cm

$z_{targ,down}$ = 75 cm

θ_{low} = 5.5 mrad

θ_{high} = 17 mrad

R_{inner} = 3.658 cm

R_{outer} = 11.306 cm

From center:

$\theta_{low,cen}$ = 6.2 mrads

$\theta_{high,cen}$ = 19.2 mrads

From downstream:

$\theta_{low,down}$ = 7.1 mrads

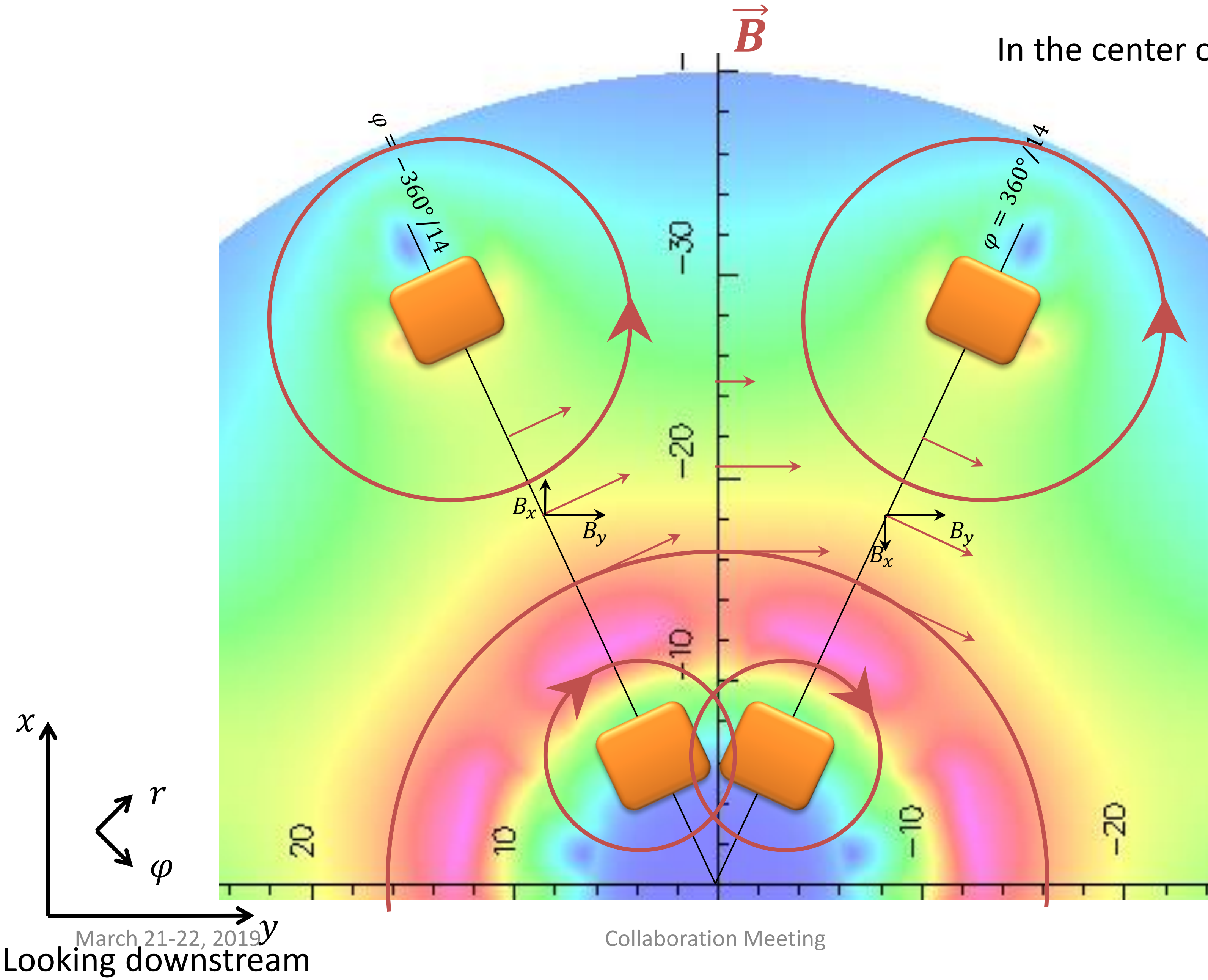
$\theta_{high,down}$ = 21 mrads

Shape of the field in a septant

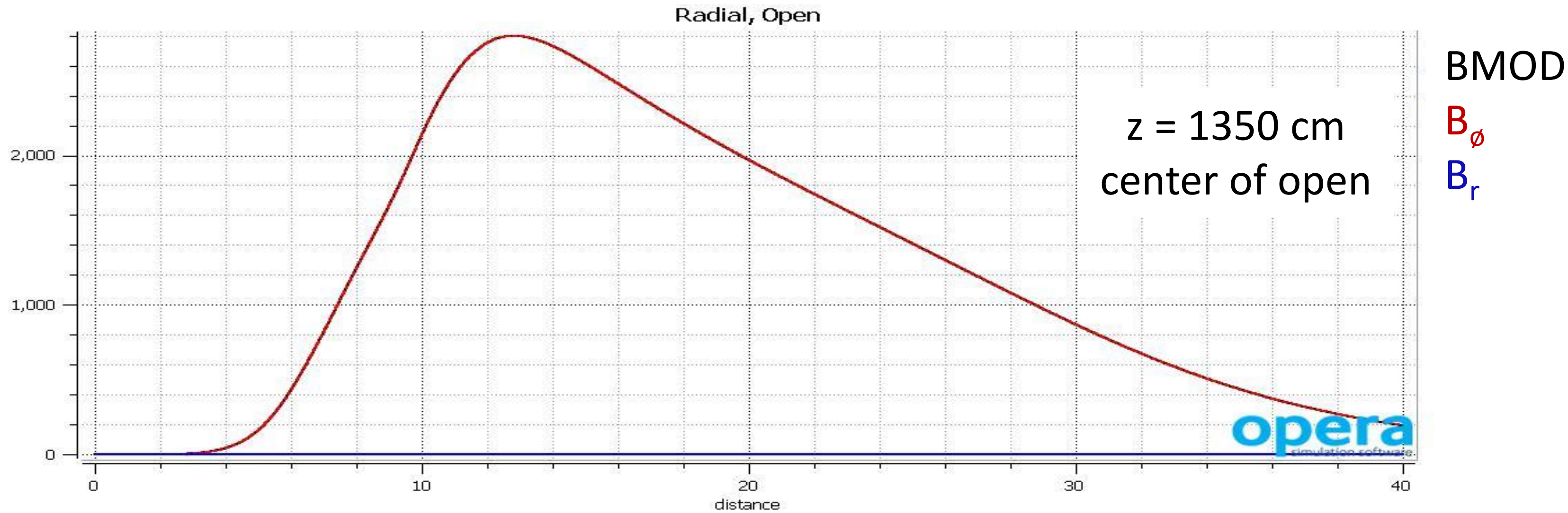
In the center of this septant:

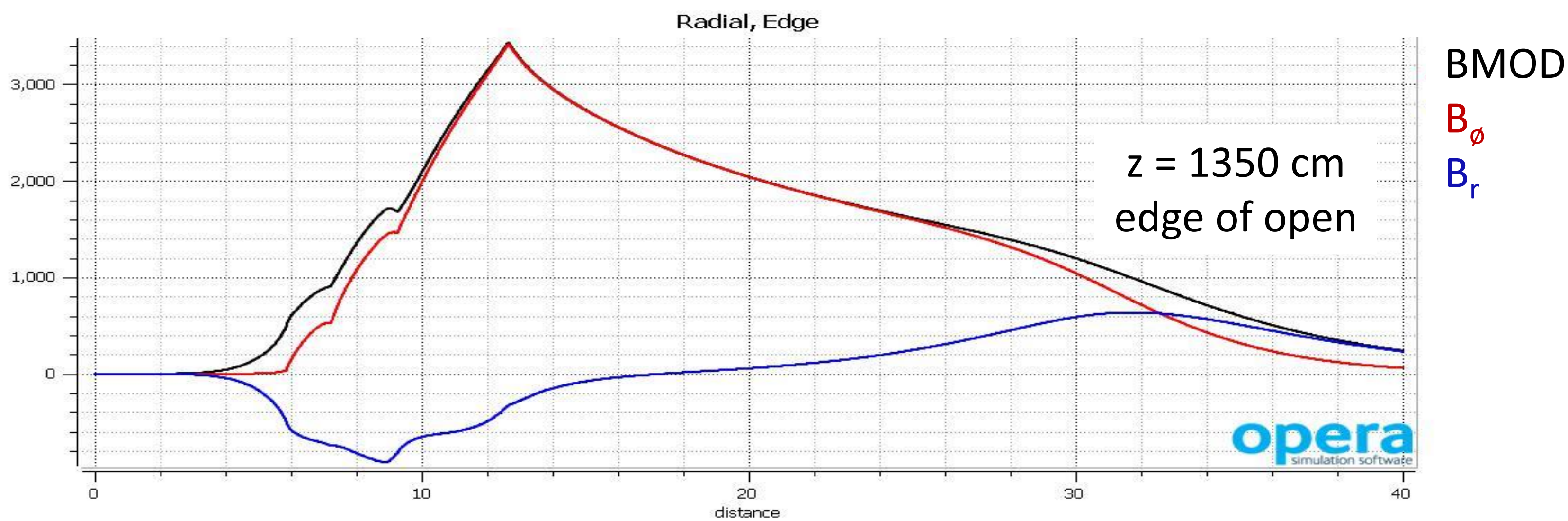
$B_r \sim B_x$

$B_\varphi \sim B_y$

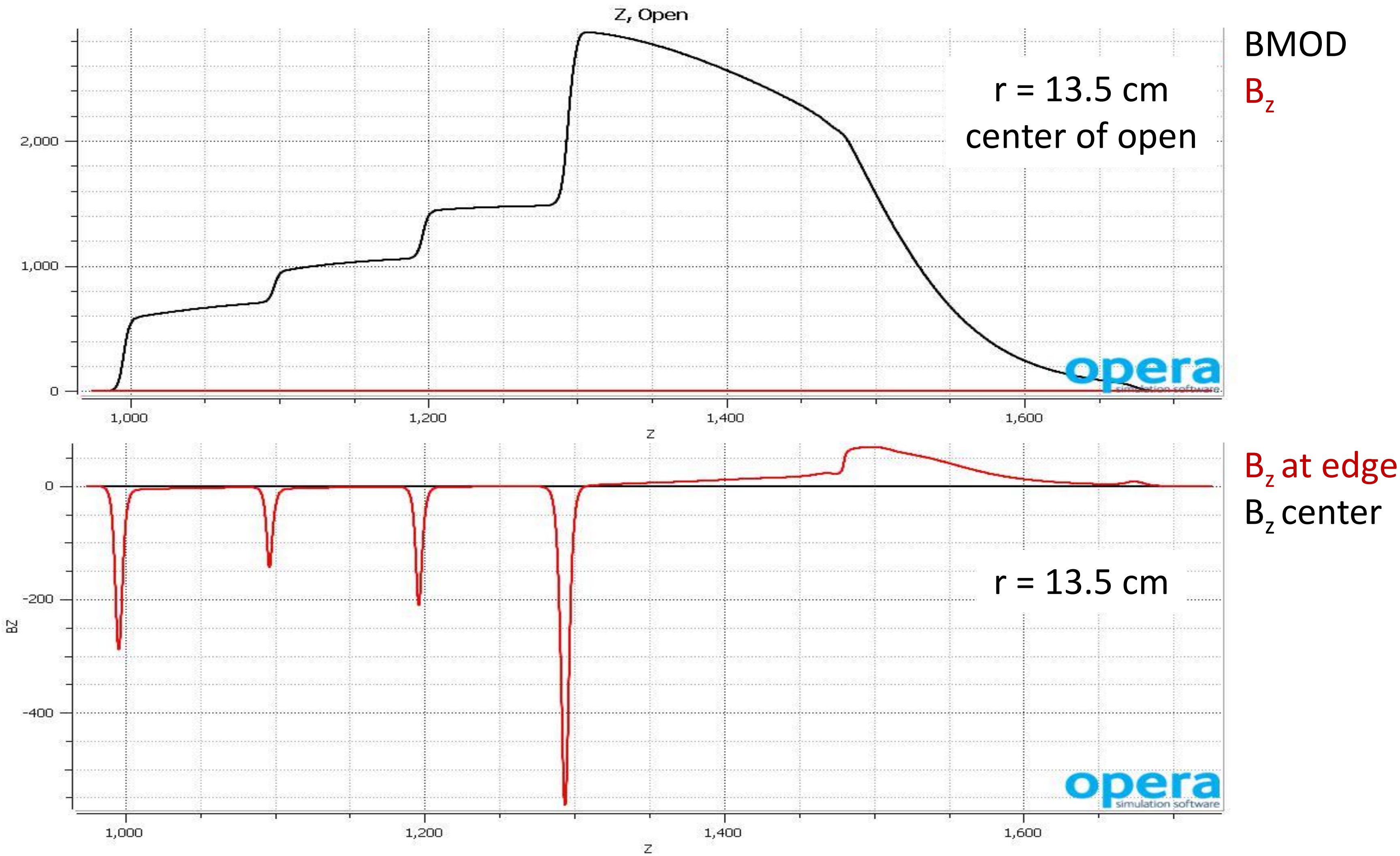


Field components vs. radius

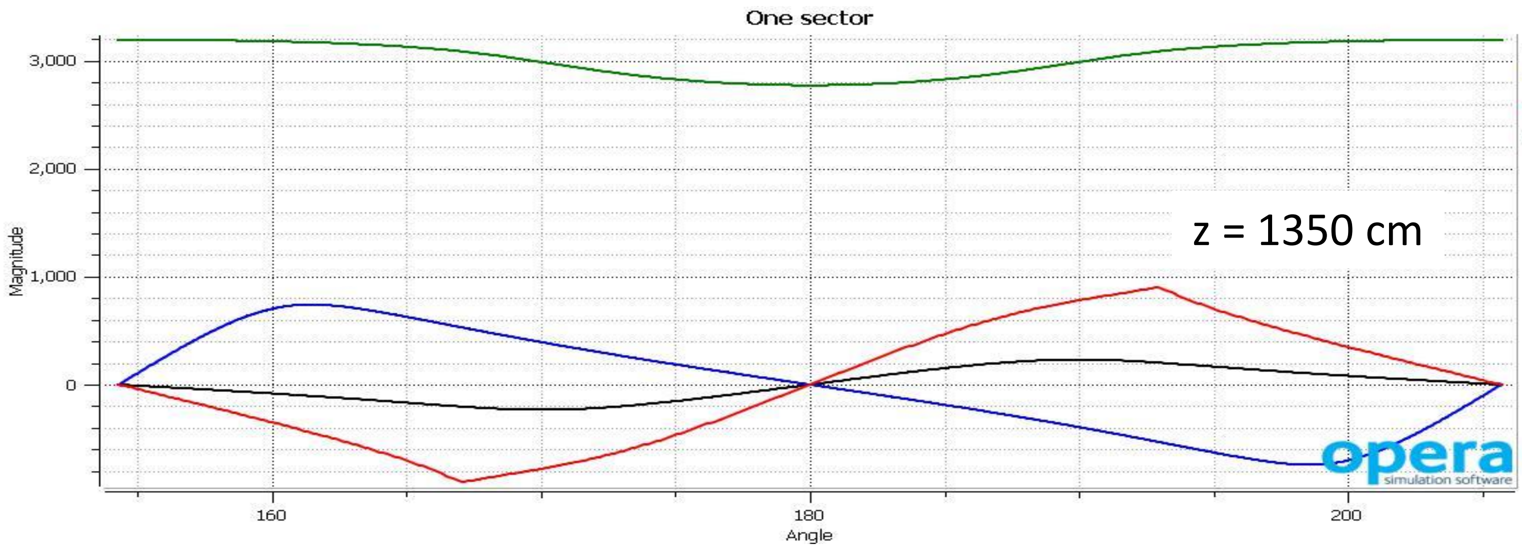


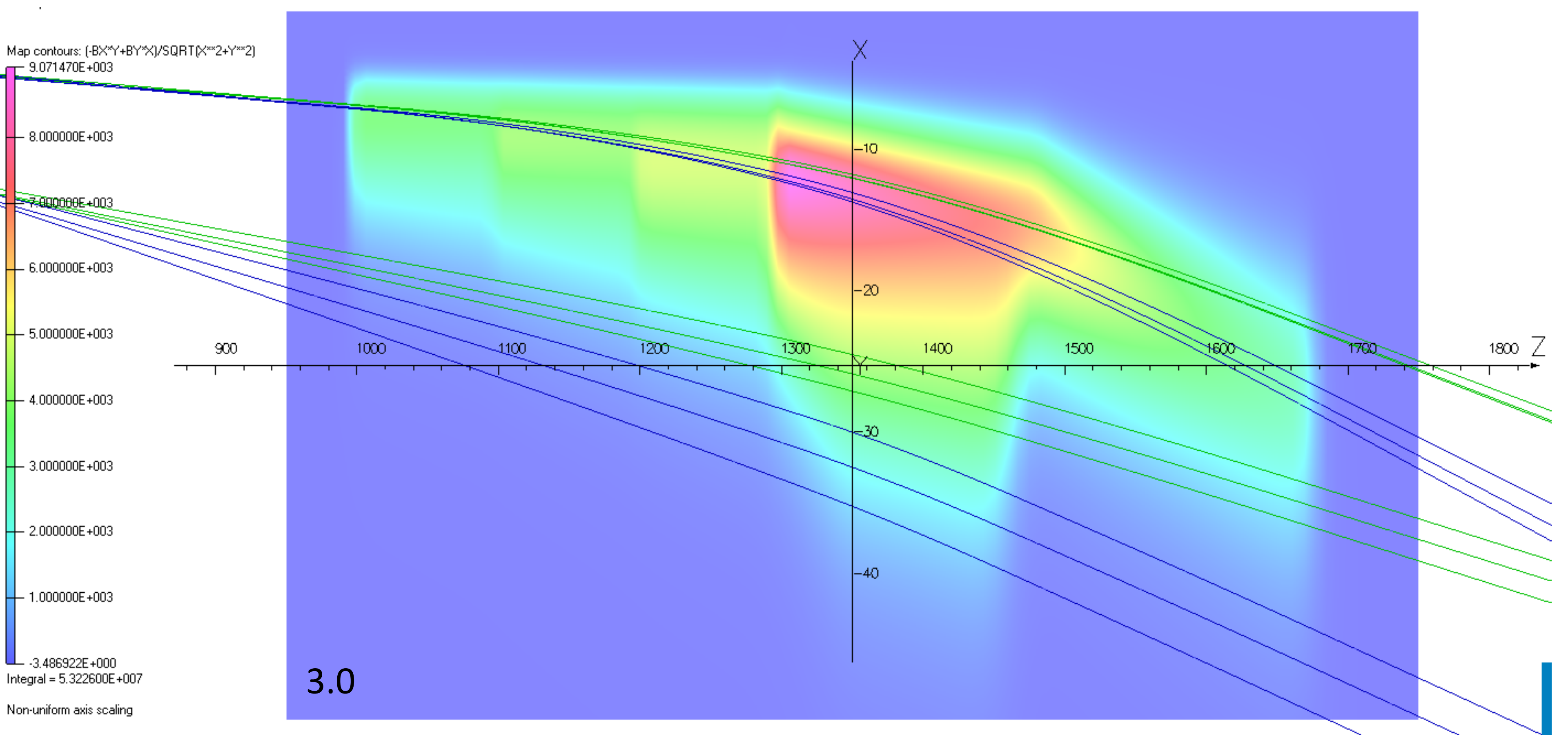


Field components vs. z



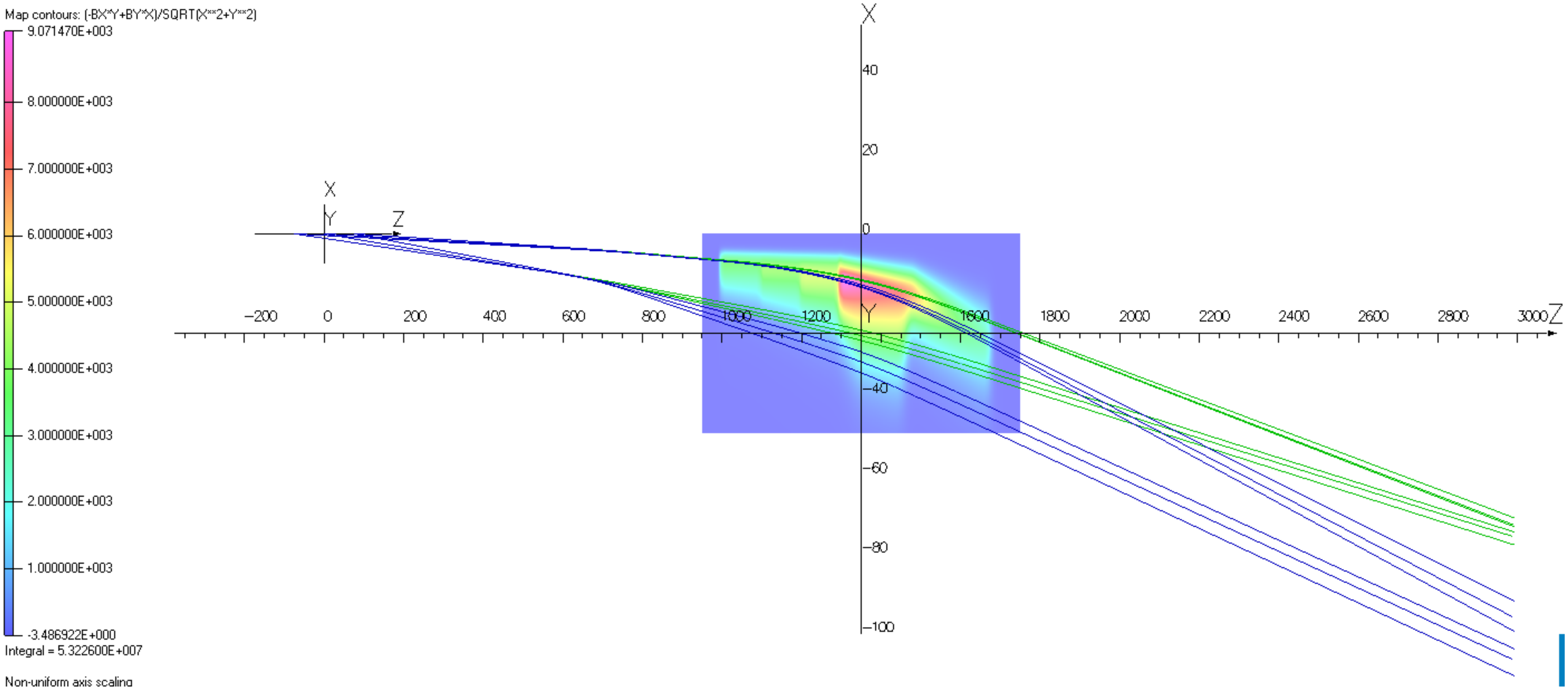
Field components vs. azimuthal angle





up (z0 =-75 cm) 5.5 and 15 mrads
middle (z0 =0 cm) 6.0 and 17 mrads
down (z0 =75 cm) 6.5 and 19 mrads
phi=0 only, near magnet

green – eps
blue - mollers



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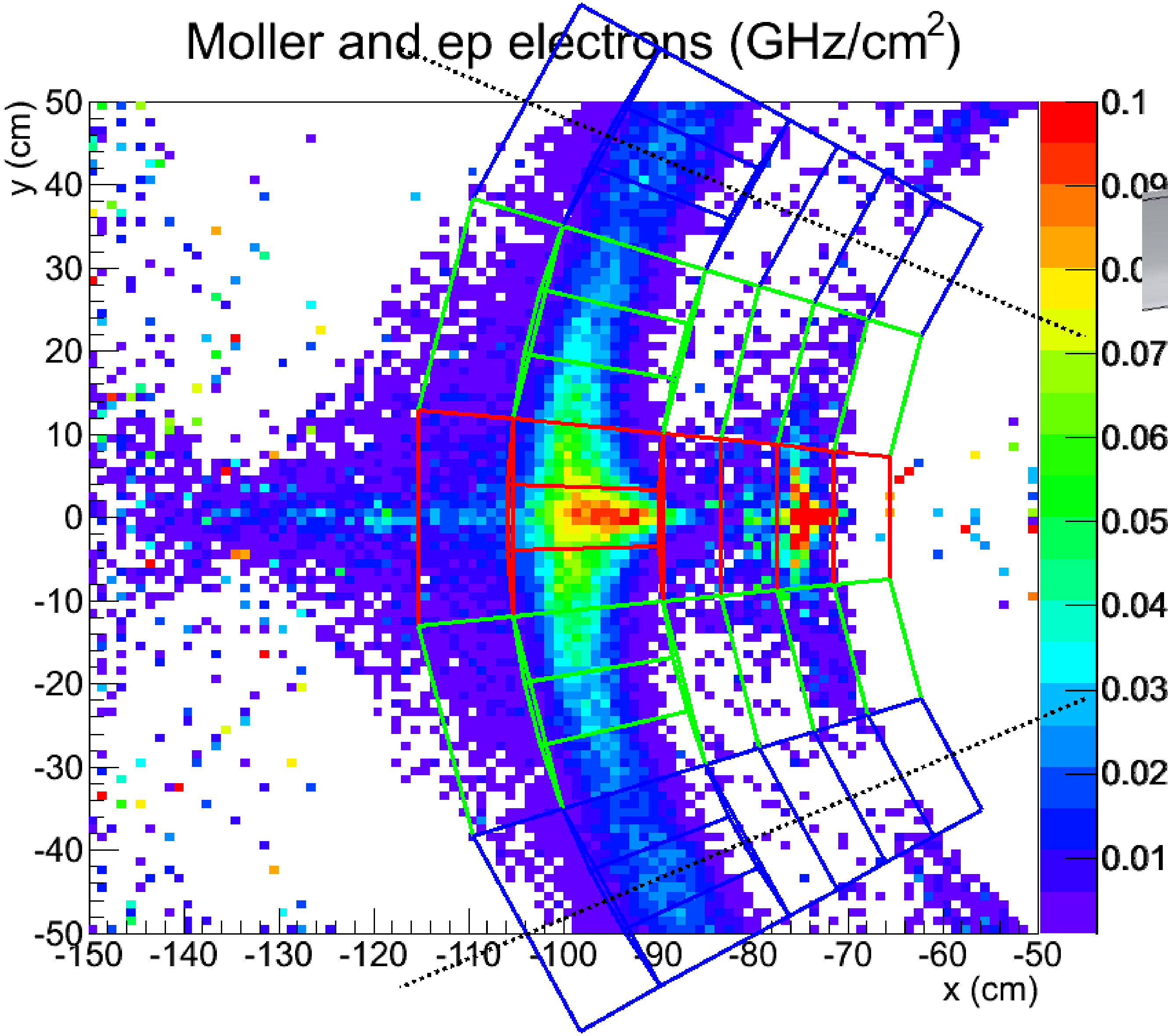
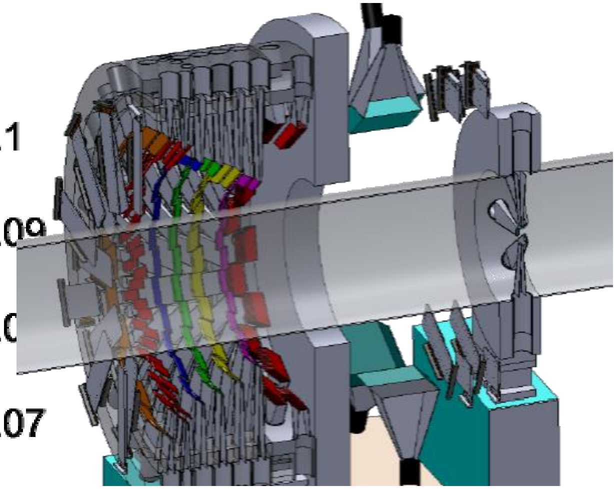
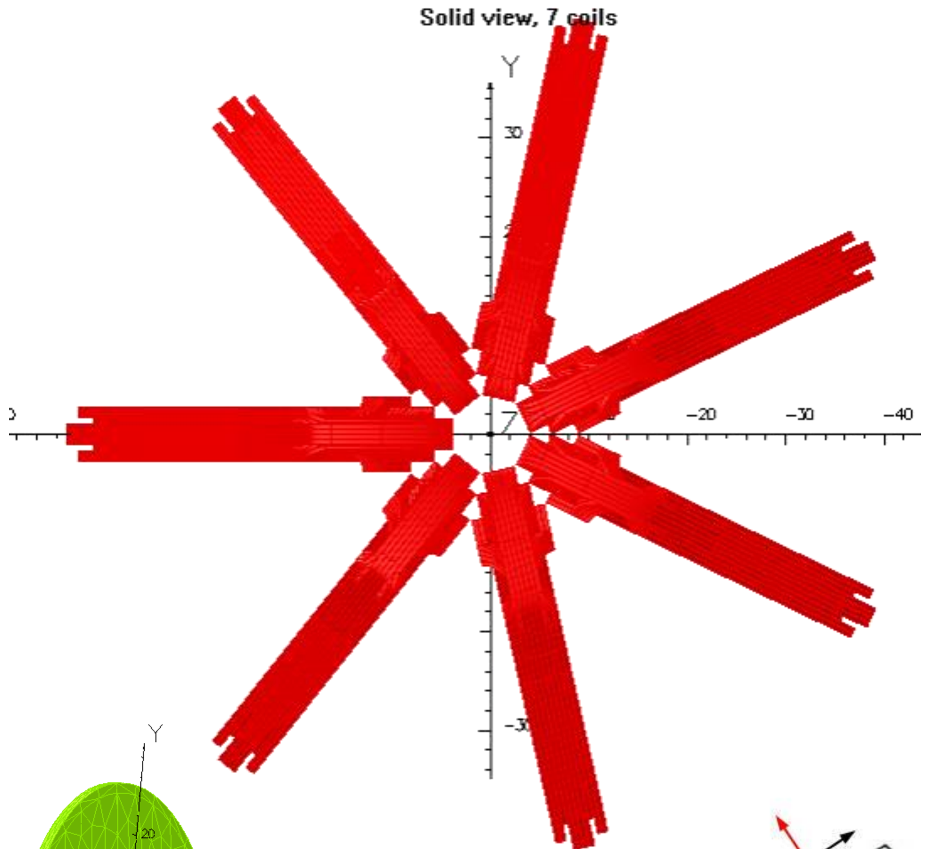
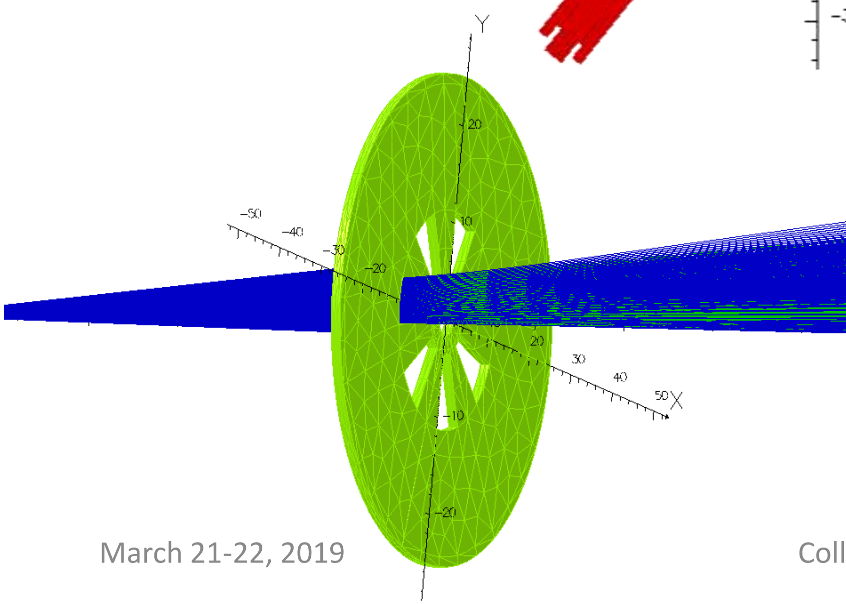


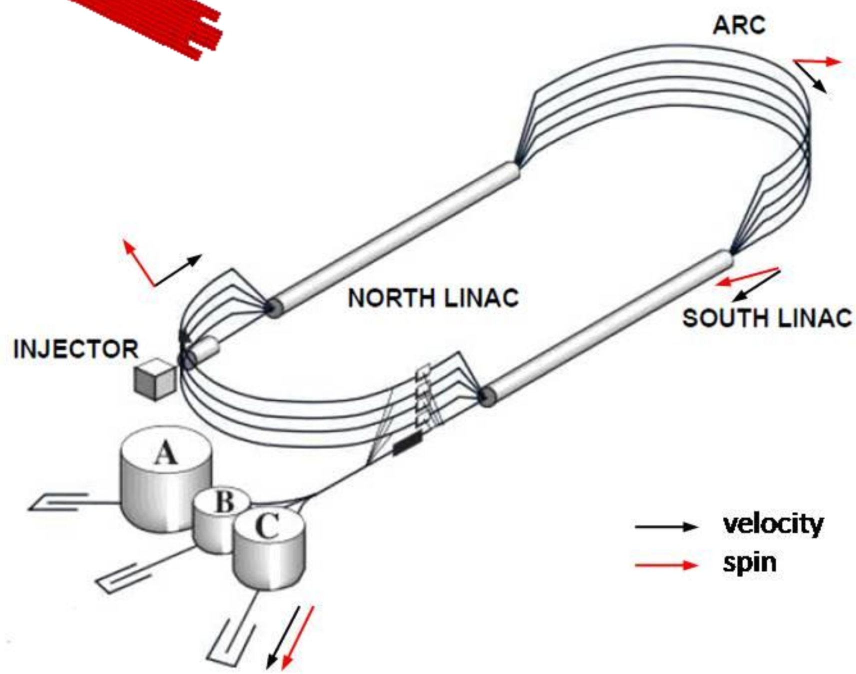


(Rate weighted 1x1cm² bins)

Sector Orientation







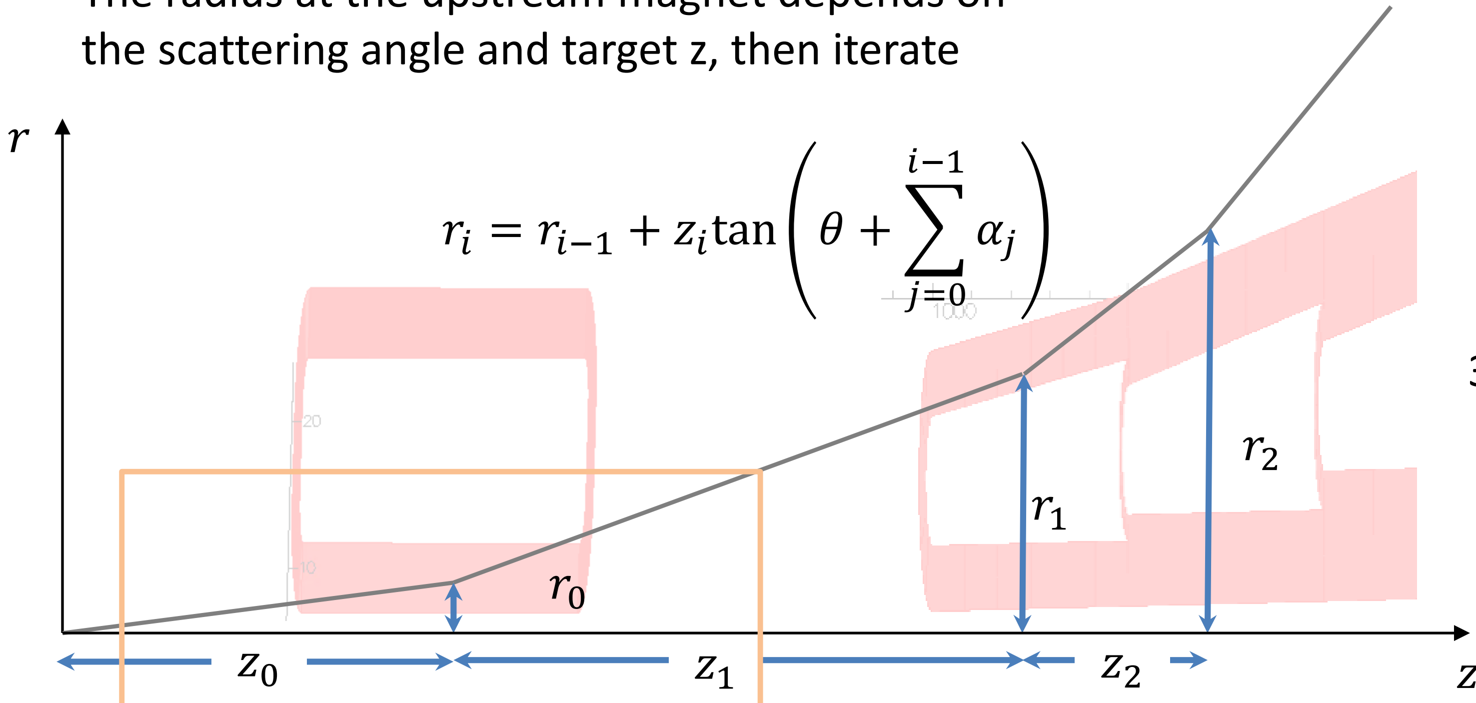
Back of the envelop calculations

(n-dimensional envelop)

- Each segment gives a "kick" at the central z location
- Field integral depends on radius of the track in that segment and the length of the segment
- Radius in a given segment depends on fields of upstream magnet segments
- The radius at the upstream magnet depends on the scattering angle and target z, then iterate

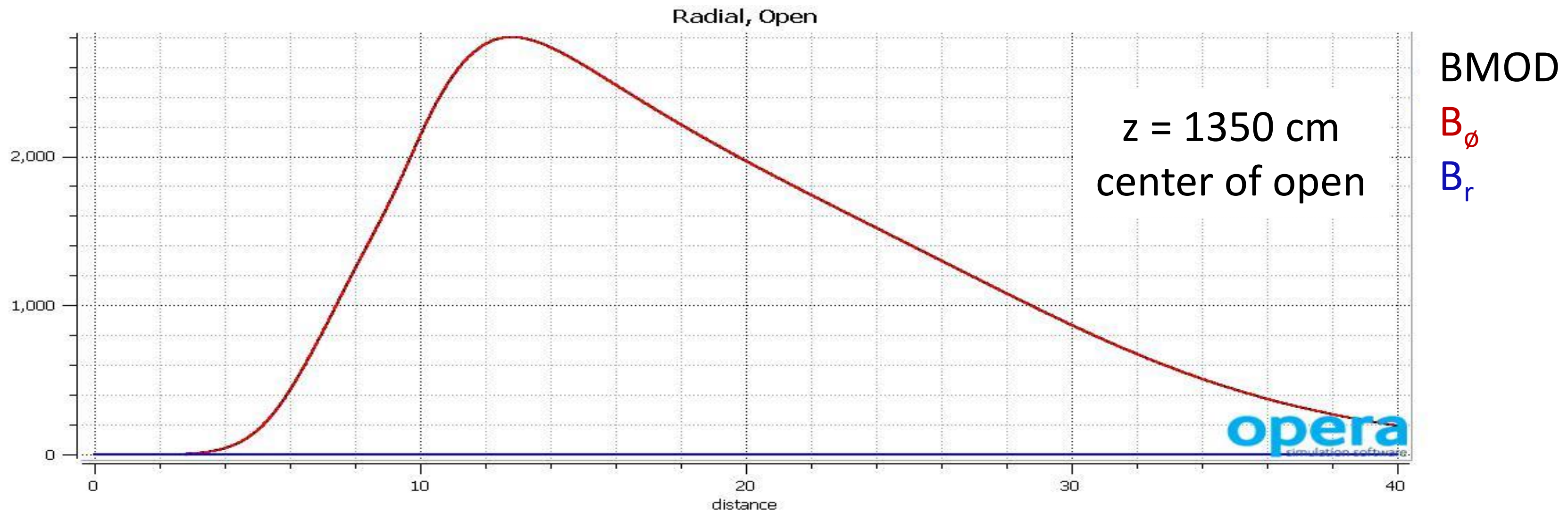
$$\alpha[rad] = \frac{\int \vec{B}\cdot d\vec{\ell}\ [Tm]}{3.33\ E\ [GeV]}$$

$$= \frac{B_{\varphi,i}(r)[T]\Delta L_i[m]}{3.33\ E_{particle}\ [GeV]}$$

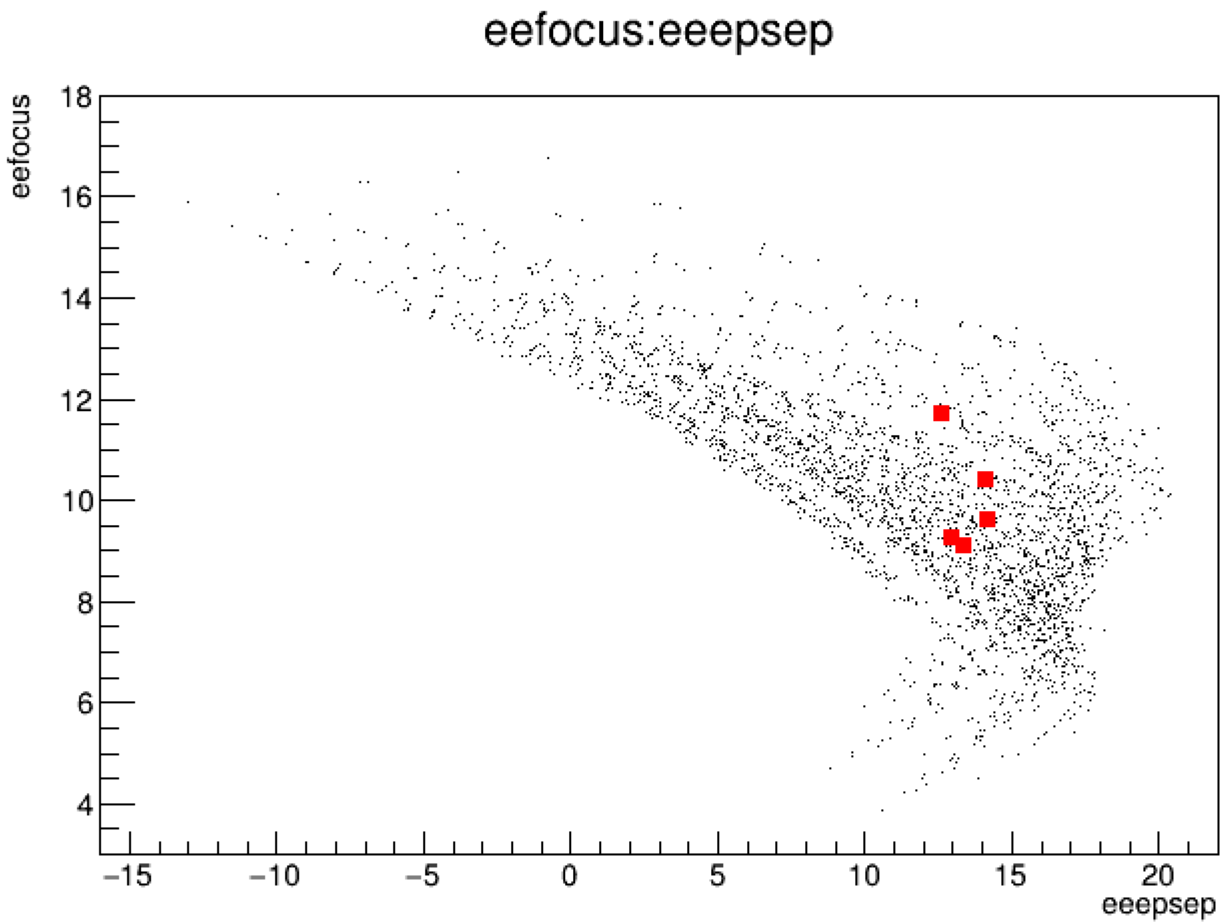


1. Get $B_{\varphi,i}(r)$ from TOSCA
2. Calculate α
3. Get r in next segment
4. Drift to detector

Field components



Phase space ee focus vs. eeepsep

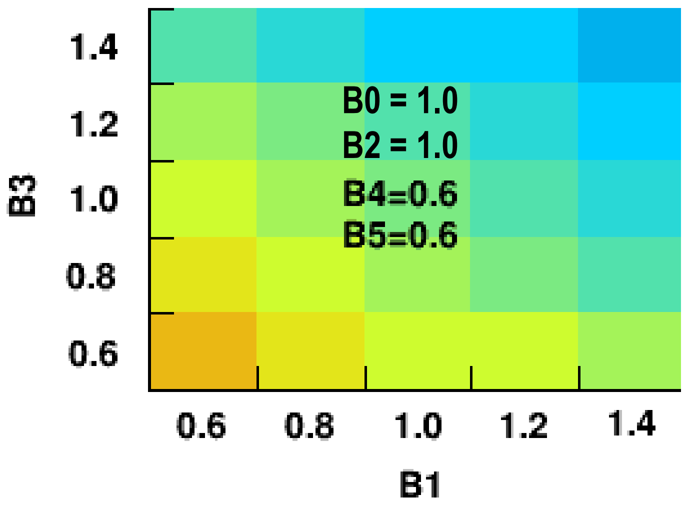


Exploring the parameter space

Plot field factor of one segment vs. field factor of another segment and weight by the quantity of interest

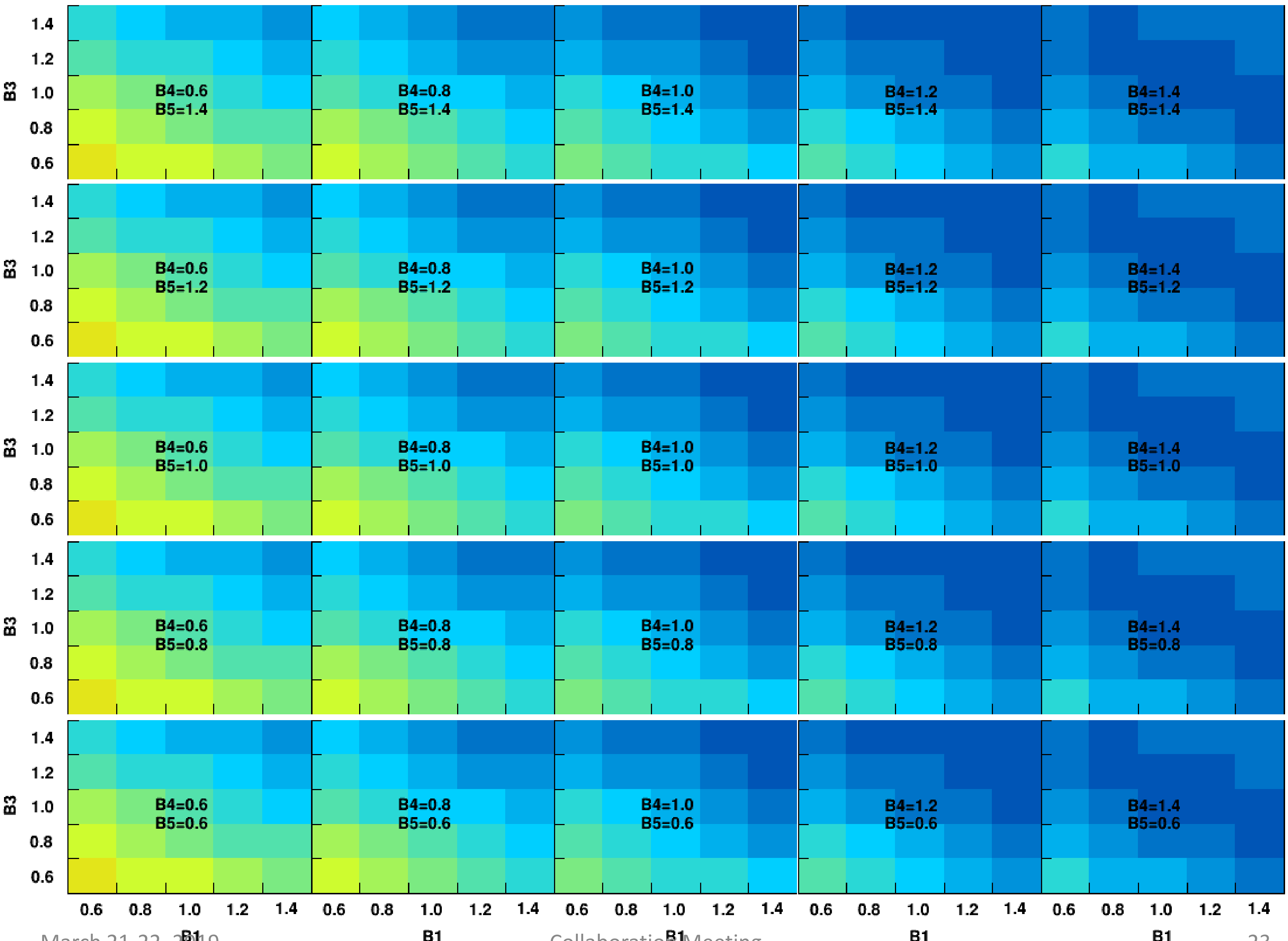
$5^6 = 15625$ combinations

B2=1.0 because it is very shallow
Reduces the number of plots to show



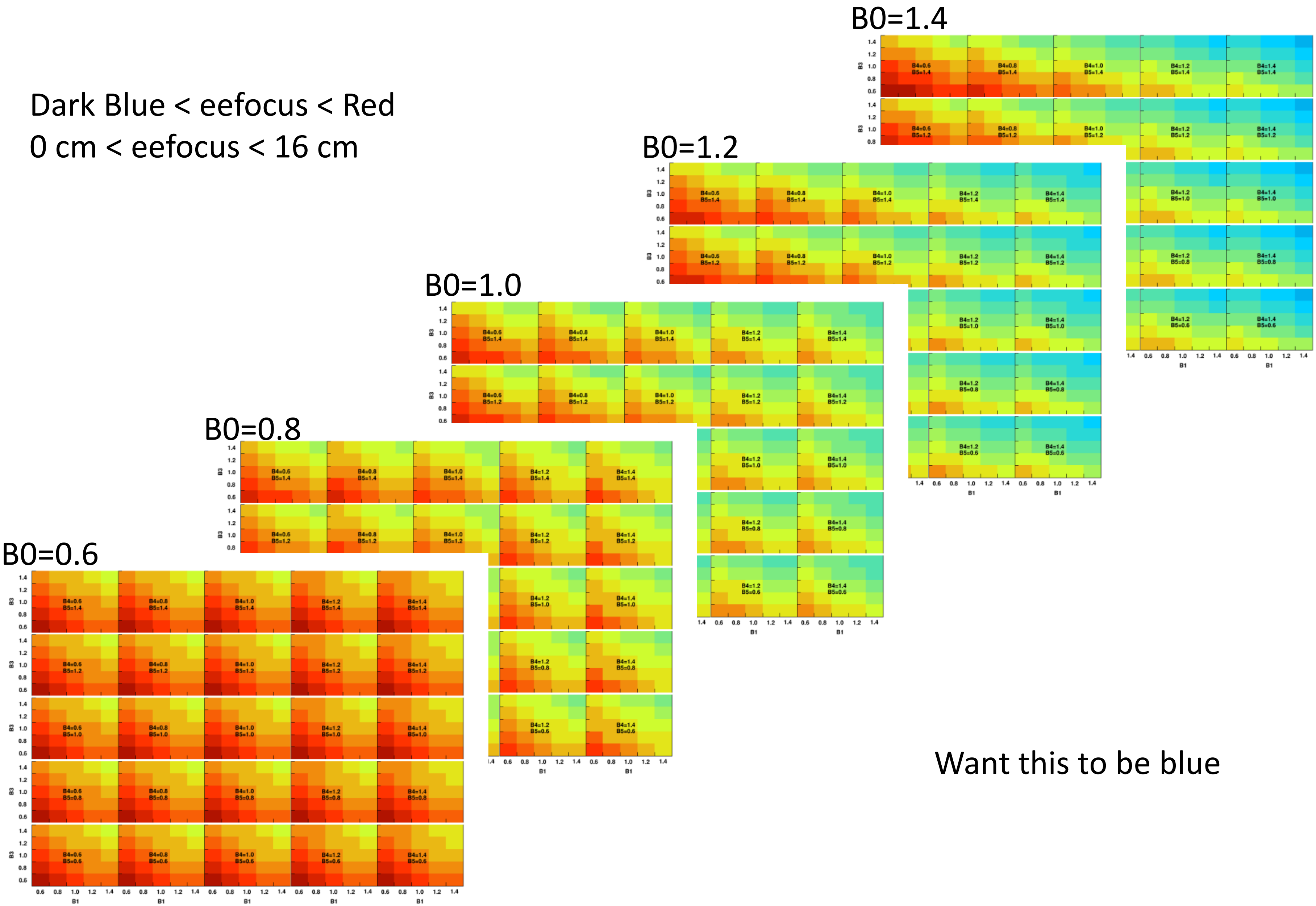
Dark Blue < epfocus < Red
0 cm < epfocus < 12 cm

epfocus, B0=1.0



Dark Blue < eefocus < Red

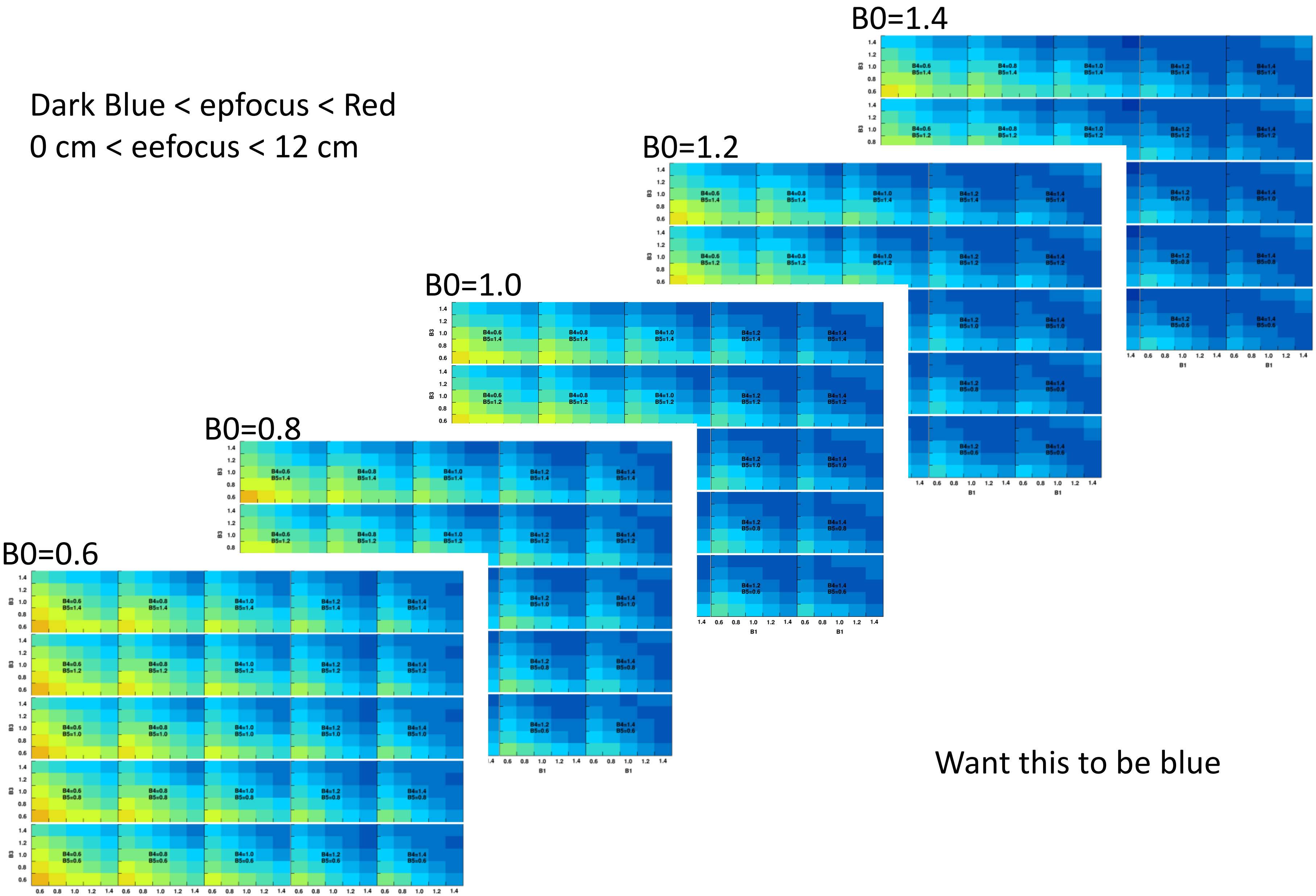
0 cm < eefocus < 16 cm



Want this to be blue

Dark Blue < epfocus < Red

0 cm < eefocus < 12 cm

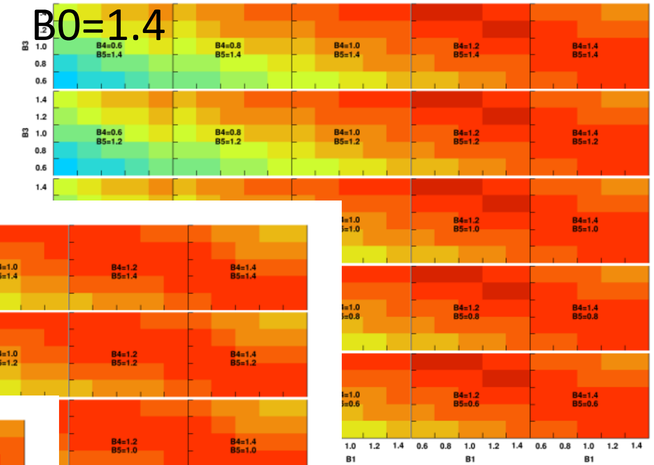


Want this to be blue

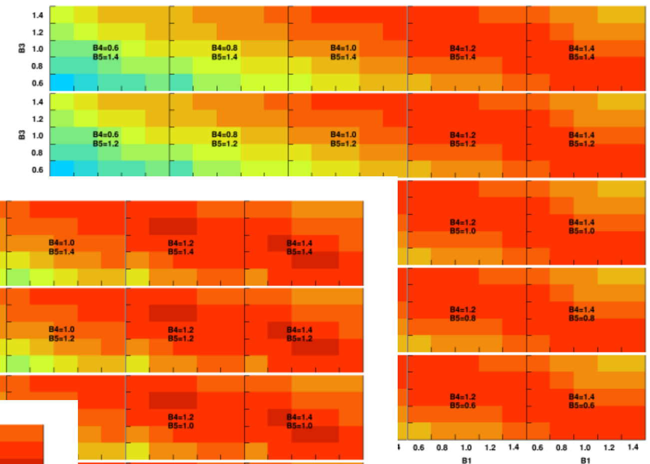
Dark Blue < eeepsep < Red

0 cm < eeepsep < 12 cm

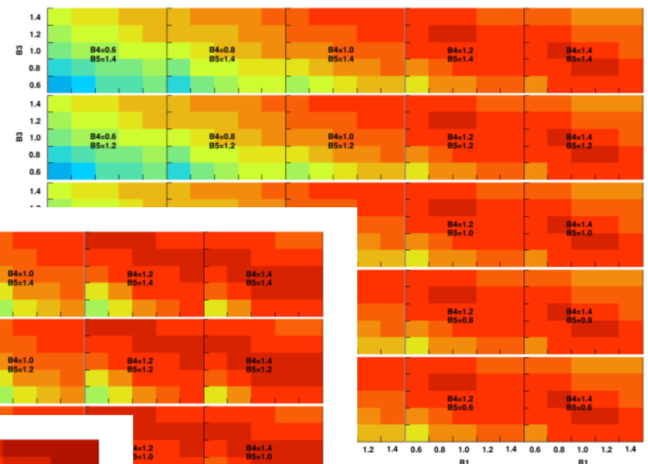
B0=1.4



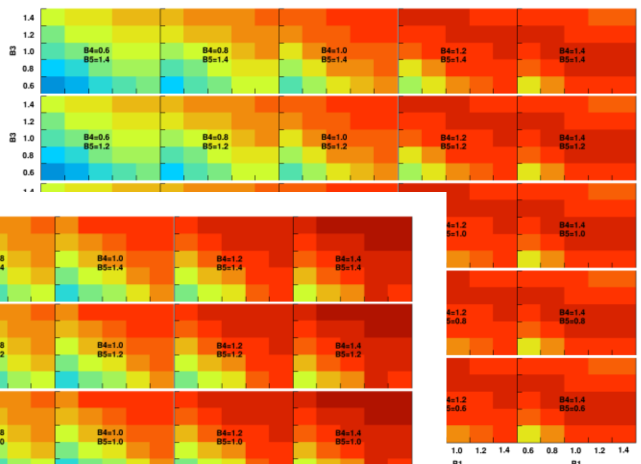
B0=1.2



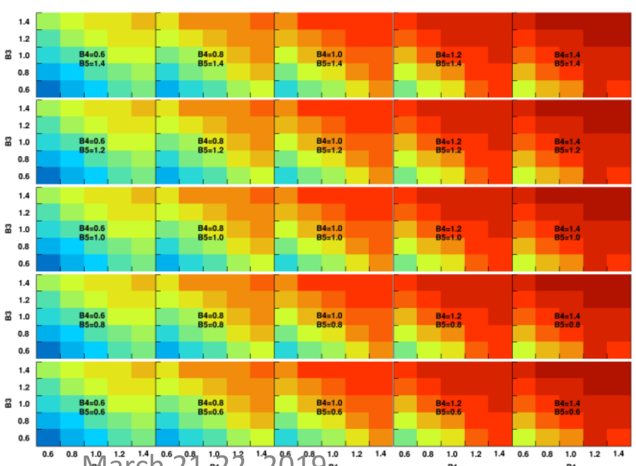
B0=1.0



B0=0.8



B0=0.6



Want this to be red

Recent Sensitivity Studies

- Effect on the optics
  – Simulations almost complete
- Interference with the envelopes
  – Look at clearances in the “slices”
- Beam steering
  – Generate field maps in beampipe
- Dose on coils
  – From both beam steering and coil offsets
- Detector Backgrounds

Sensitivity Studies

field maps for a **single coil** misplaced by 10 steps

$$\delta A_{raw}\left(\frac{\Delta\langle A_{raw}\rangle}{\Delta z}\right)^{-1} = \delta z$$

$$\delta A_{raw} = 0.1\ ppb$$

Along beam, Z – ± 0.5 cm

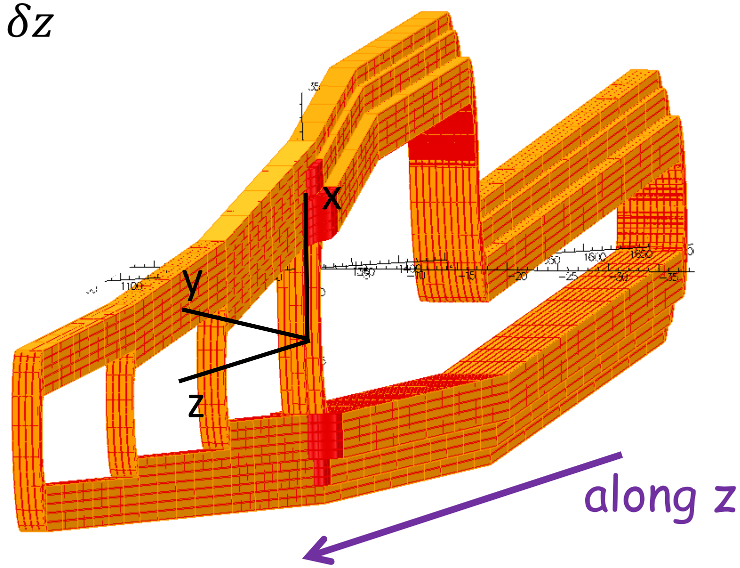
Radially, R – ± 0.5 cm

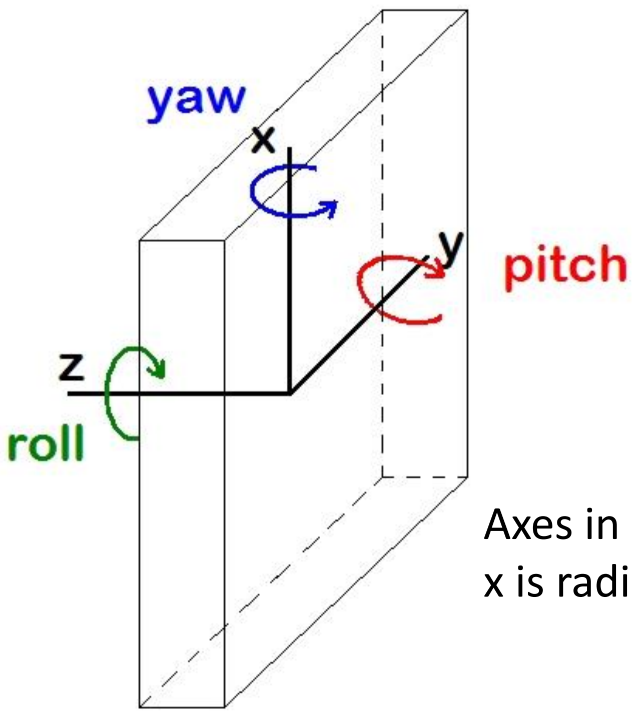
Azimuthally, T – ± 0.05°

Roll – ±0.05°

Pitch – ±0.05°

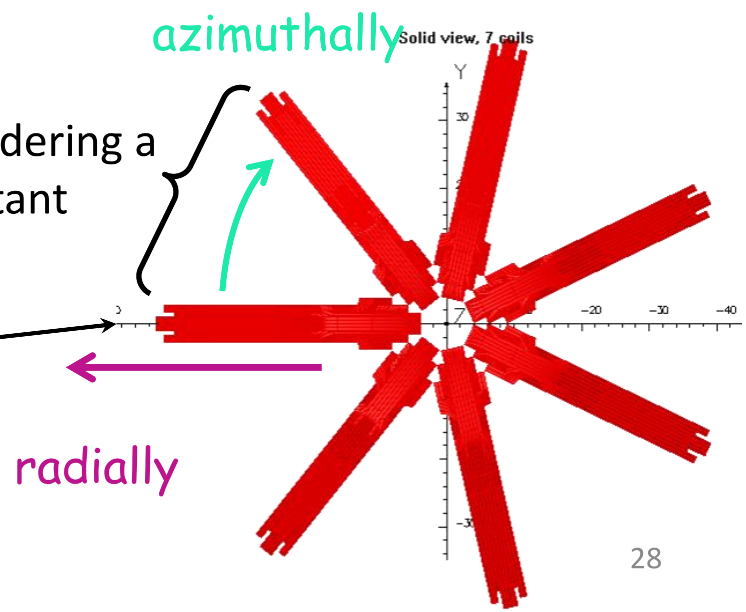
Yaw – ±0.05°



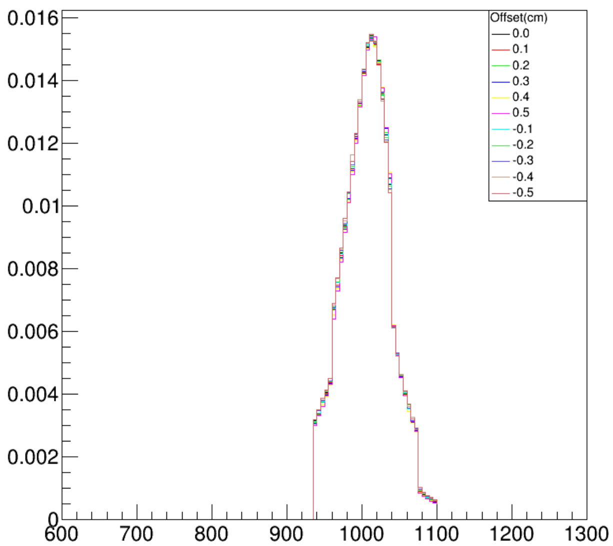


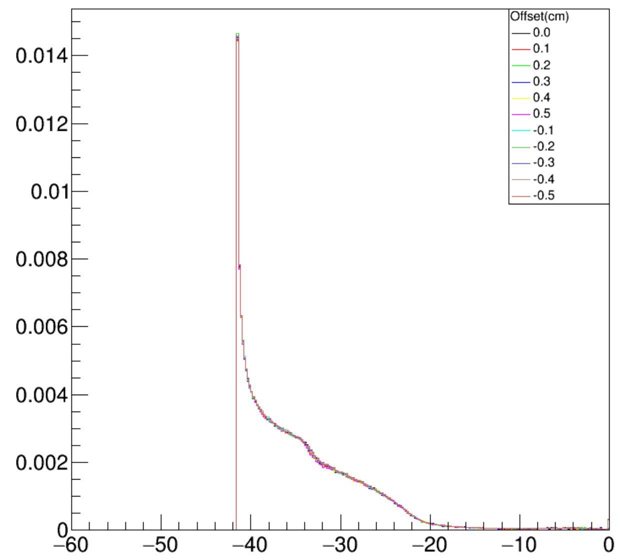
Only considering a single septant

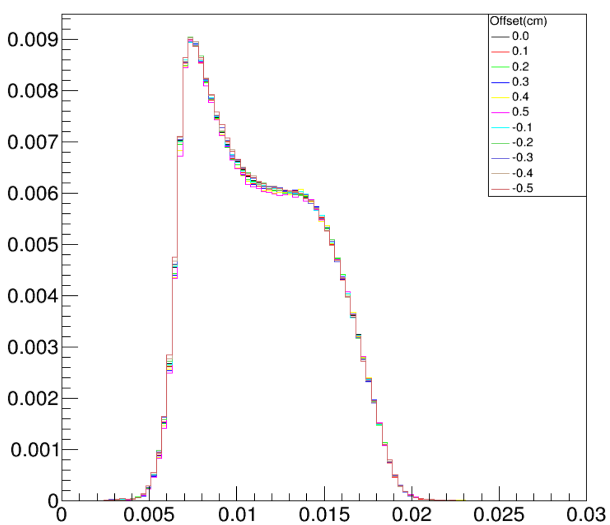
Axes in frame of single coil, x is radial direction

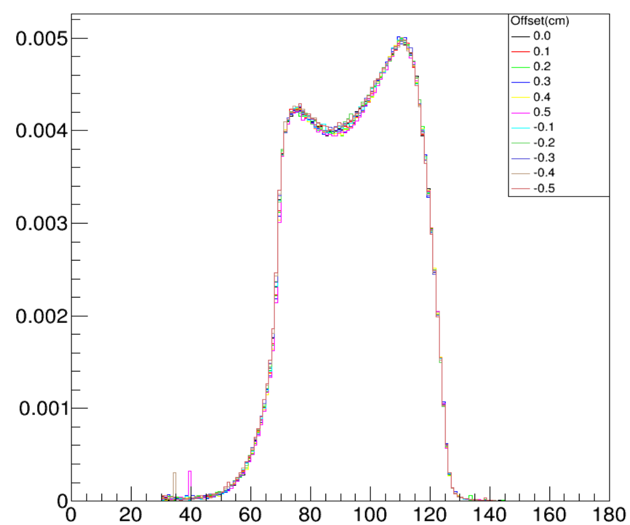


1D Distributions

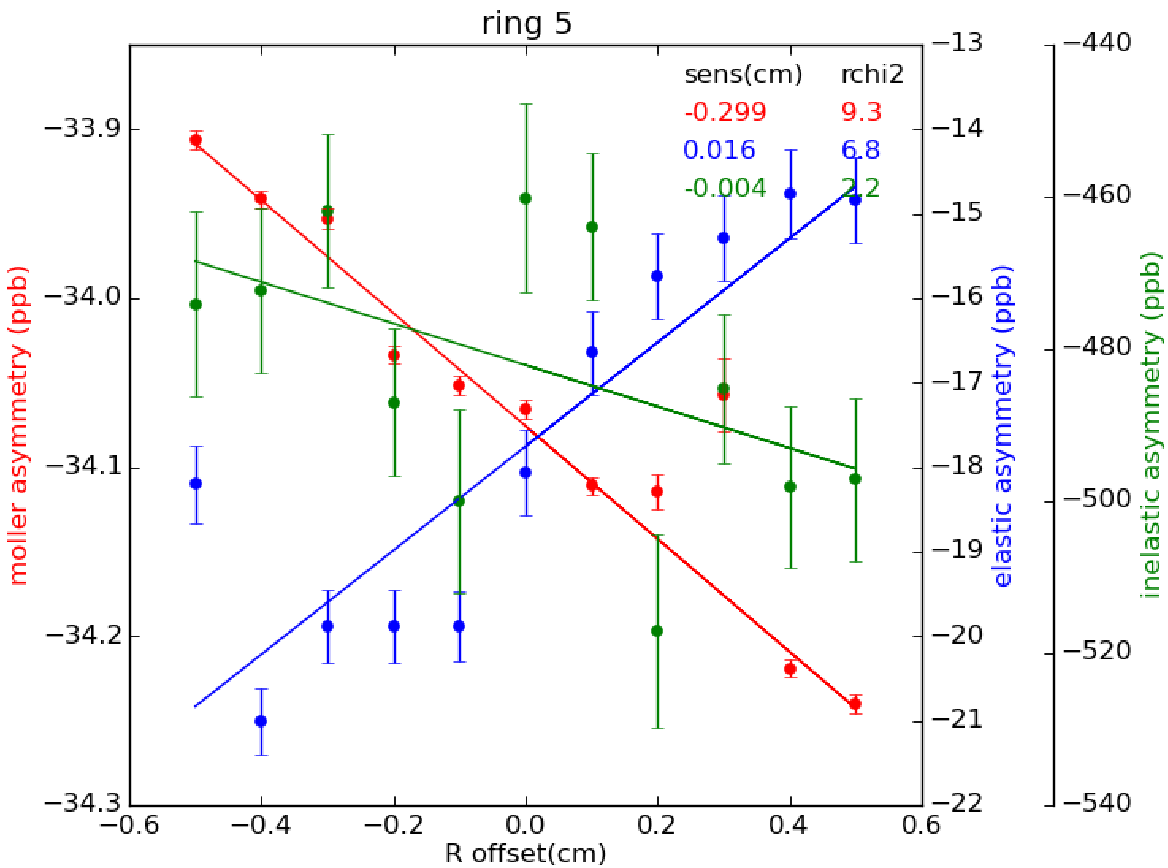








Asymmetry vs Radial Offset

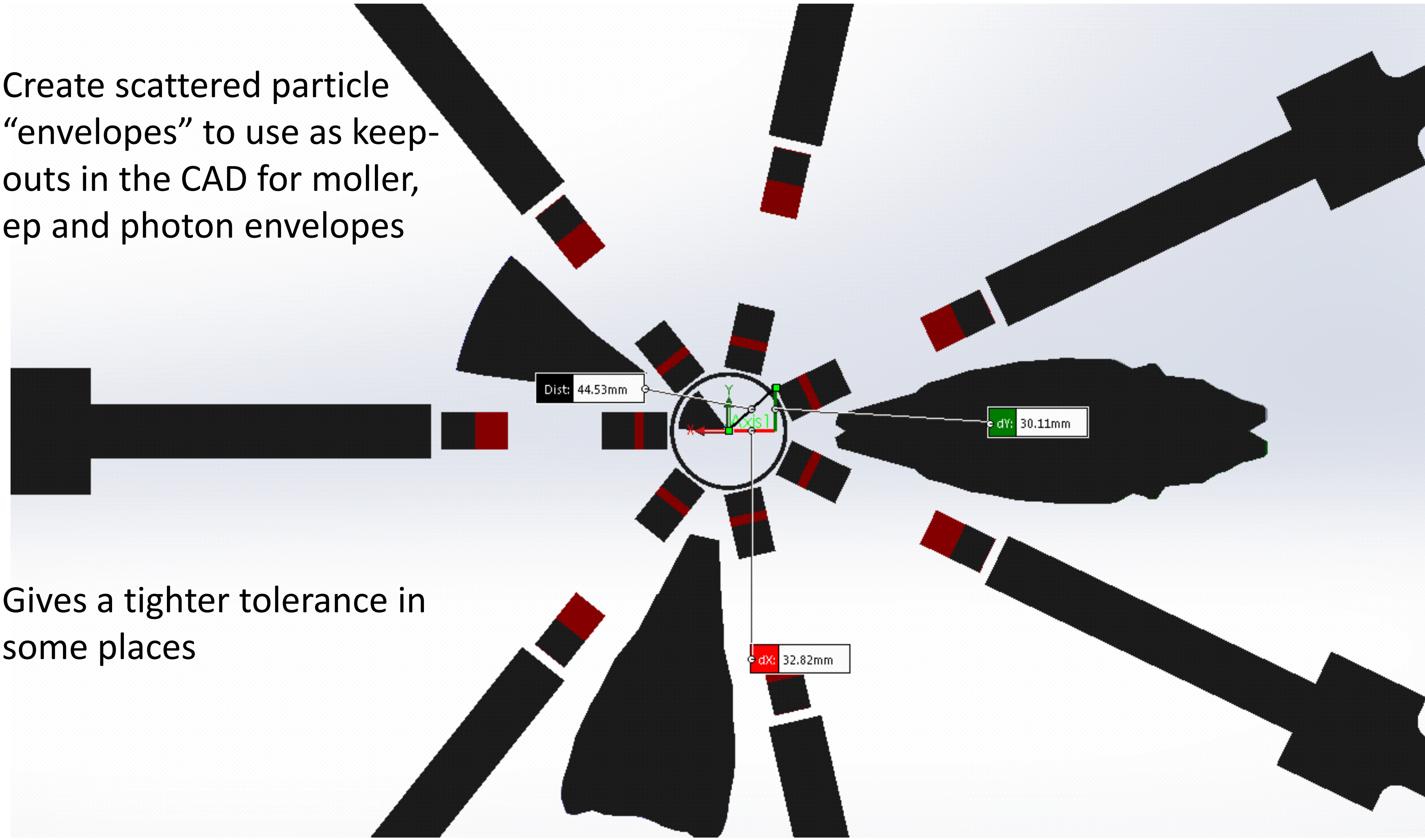


High statistics runs with up-to-date geometry confirm a tolerance of ±3 mm for the radial offset (tightest tolerance)

Other tolerances are coming out a little tighter than before but the radial offset is still the tightest

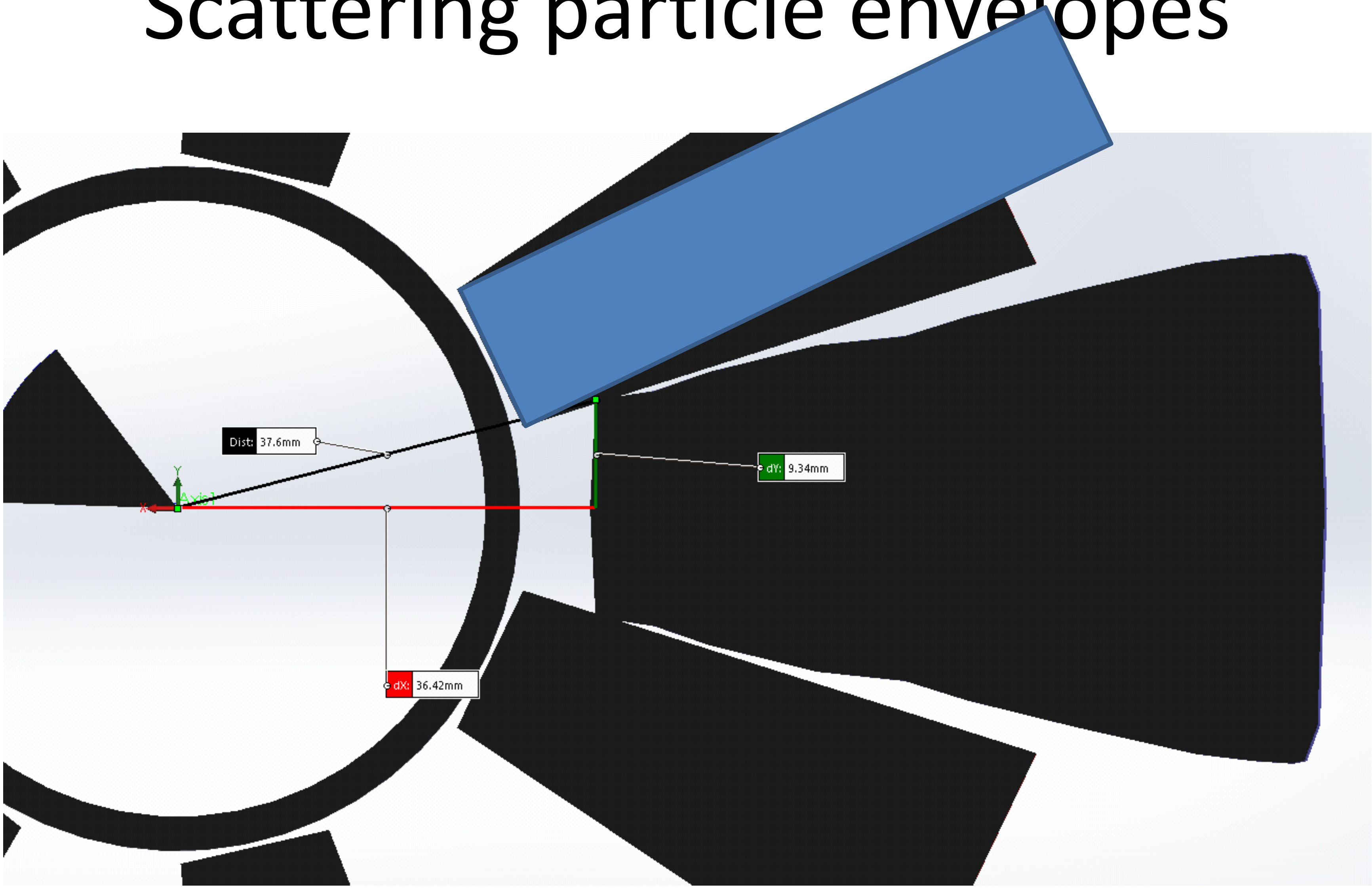
Scattered particle envelopes

Create scattered particle "envelopes" to use as keep-outs in the CAD for moller, ep and photon envelopes



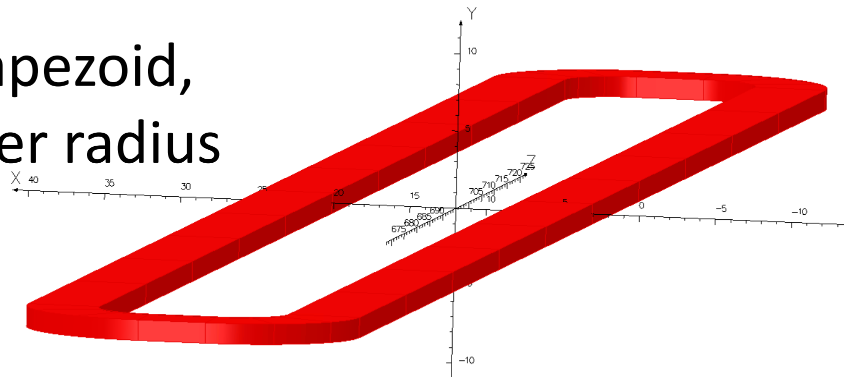
Gives a tighter tolerance in some places

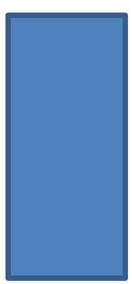
Scattering particle envelopes



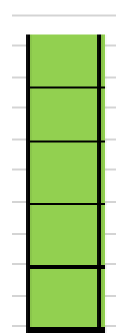
Comparisons between

- Default – trapezoidal blocky model
- Rectangle 1 – same height as trapezoid,
  same width as inner radius
- Rectangle 2 – Dave's version 10
- Rectangle 3 – Dave's version 13



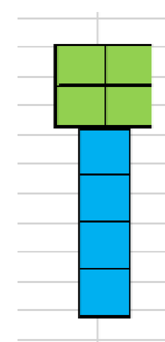


13.3 x 46.4 mm^2



10

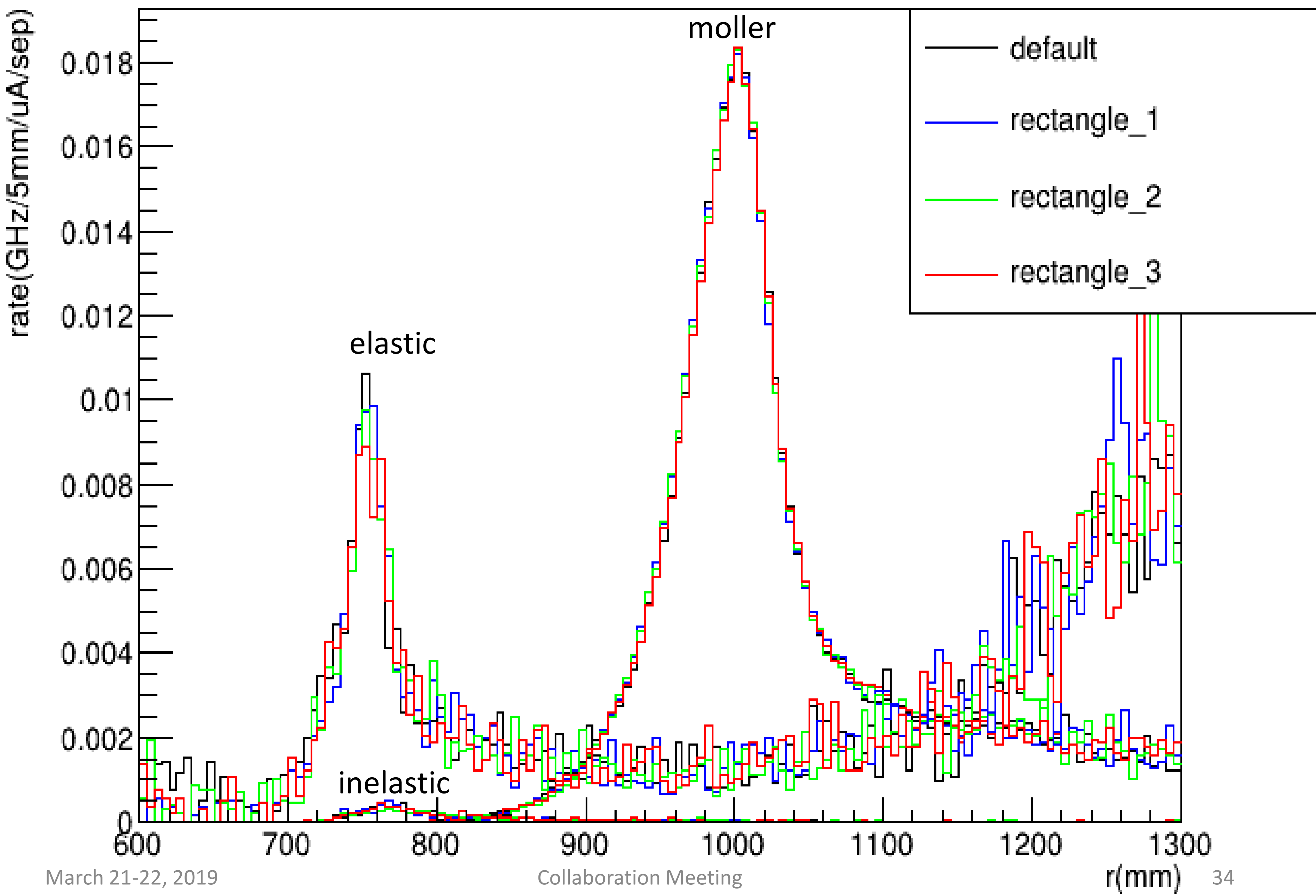
9.2x51.6 mm^2



13

15.9 or 8.2
x 46.8 mm^2

Overall Radial Distribution

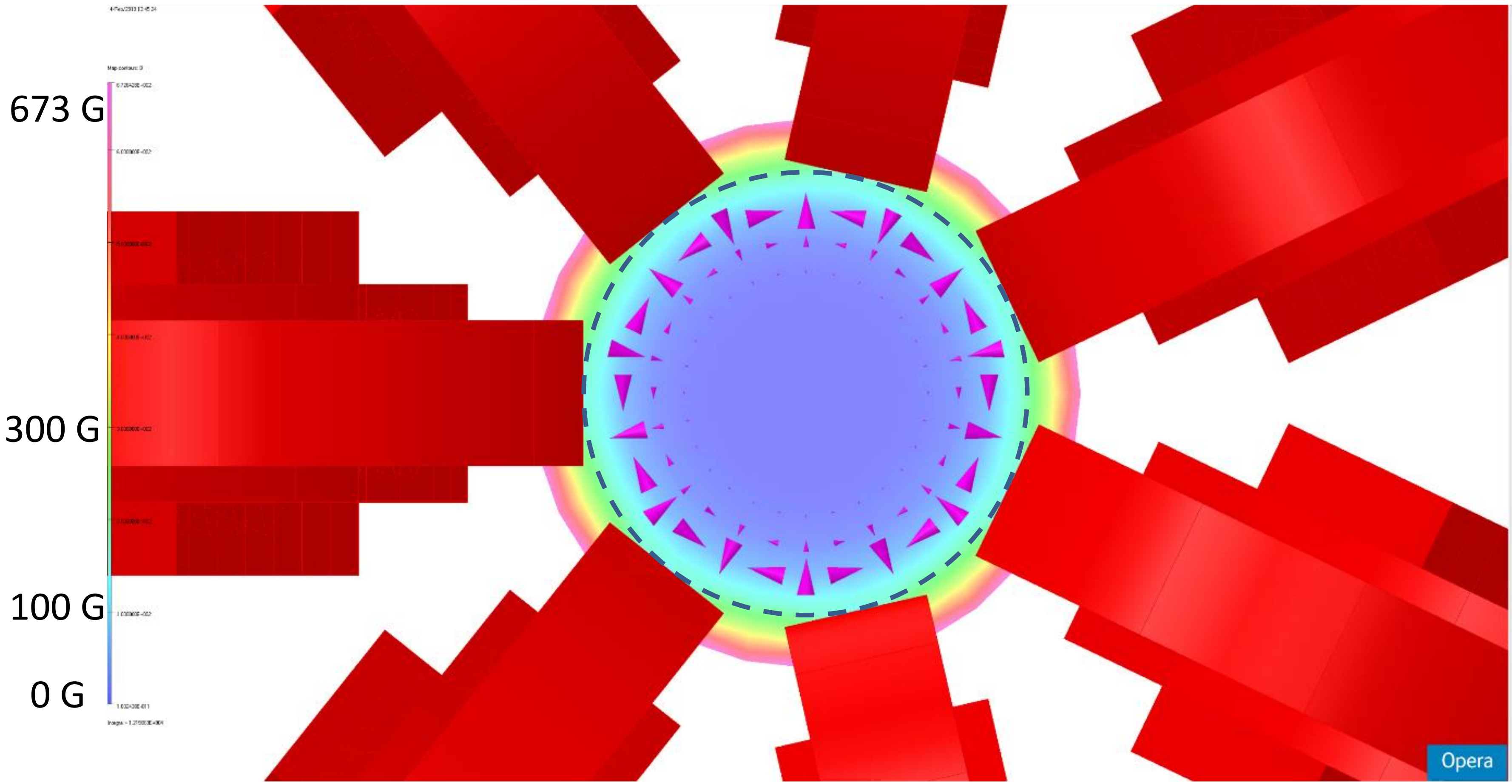


Stray Fields

r = 4.08 cm

nominal – symmetric

Max vector shown: 150 G

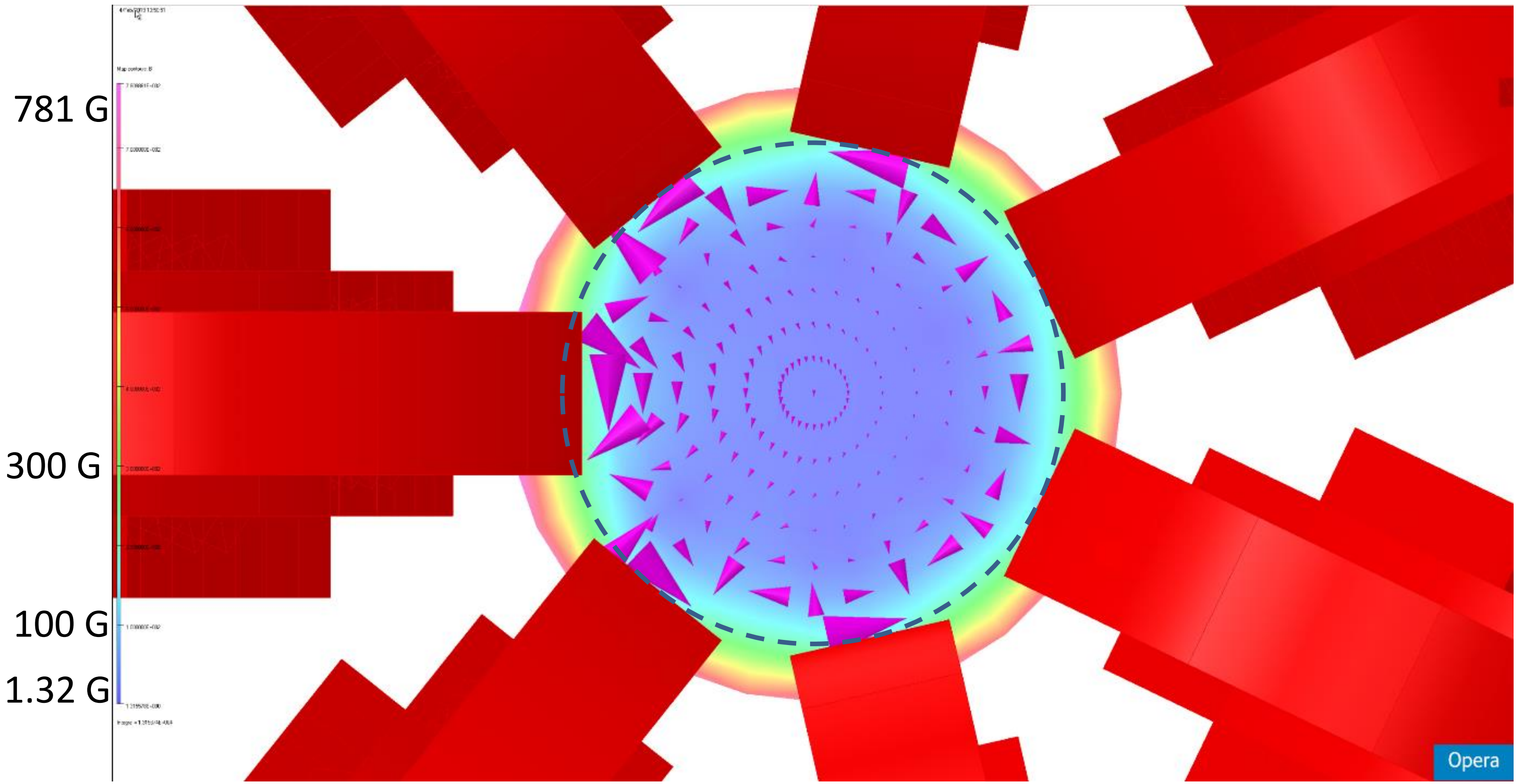


Effect of Offsets on Stray Fields

r = 4.08 cm

3mm inward – deflect right

Max vector shown: 150 G

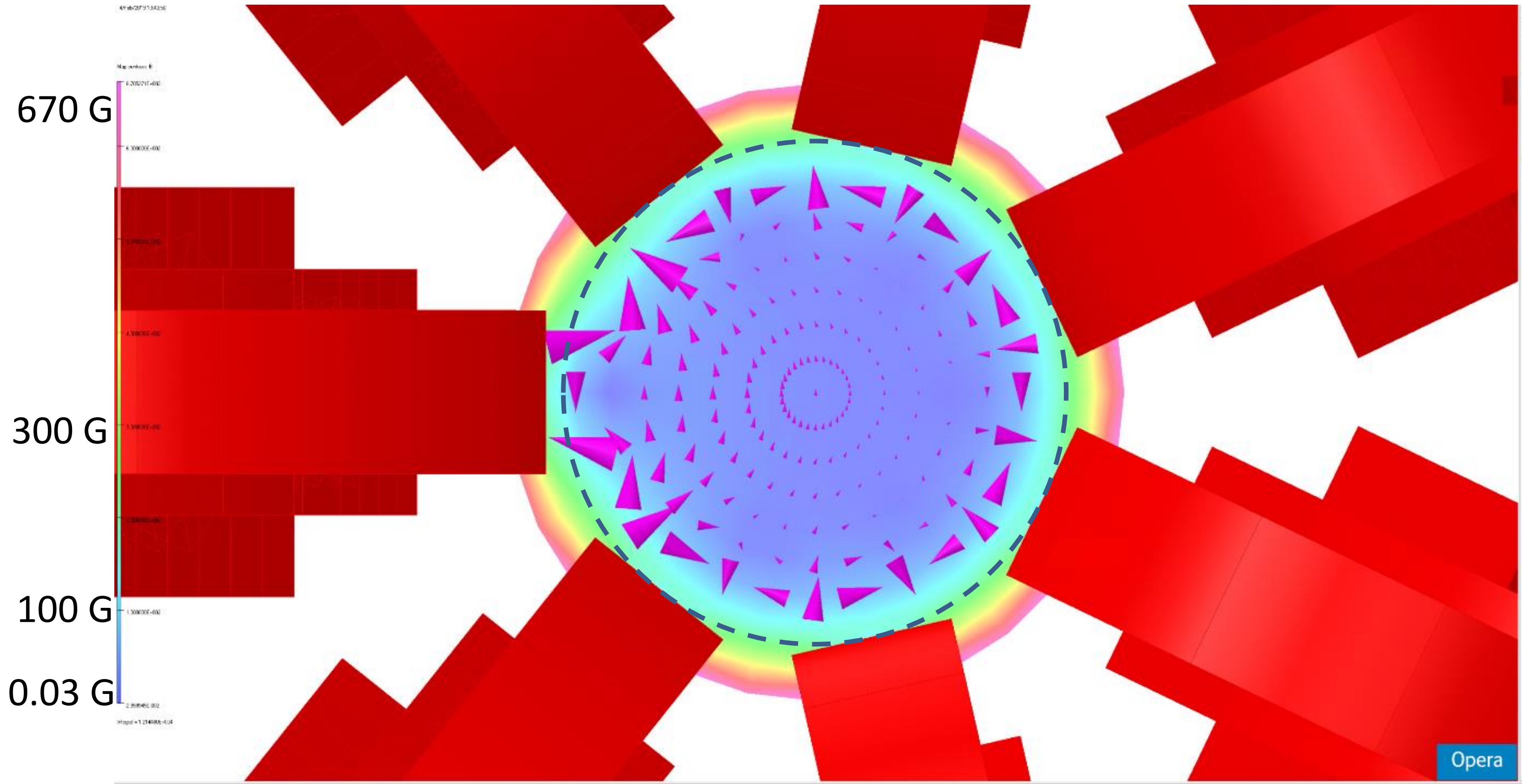


Effect of Offsets on Stray Fields

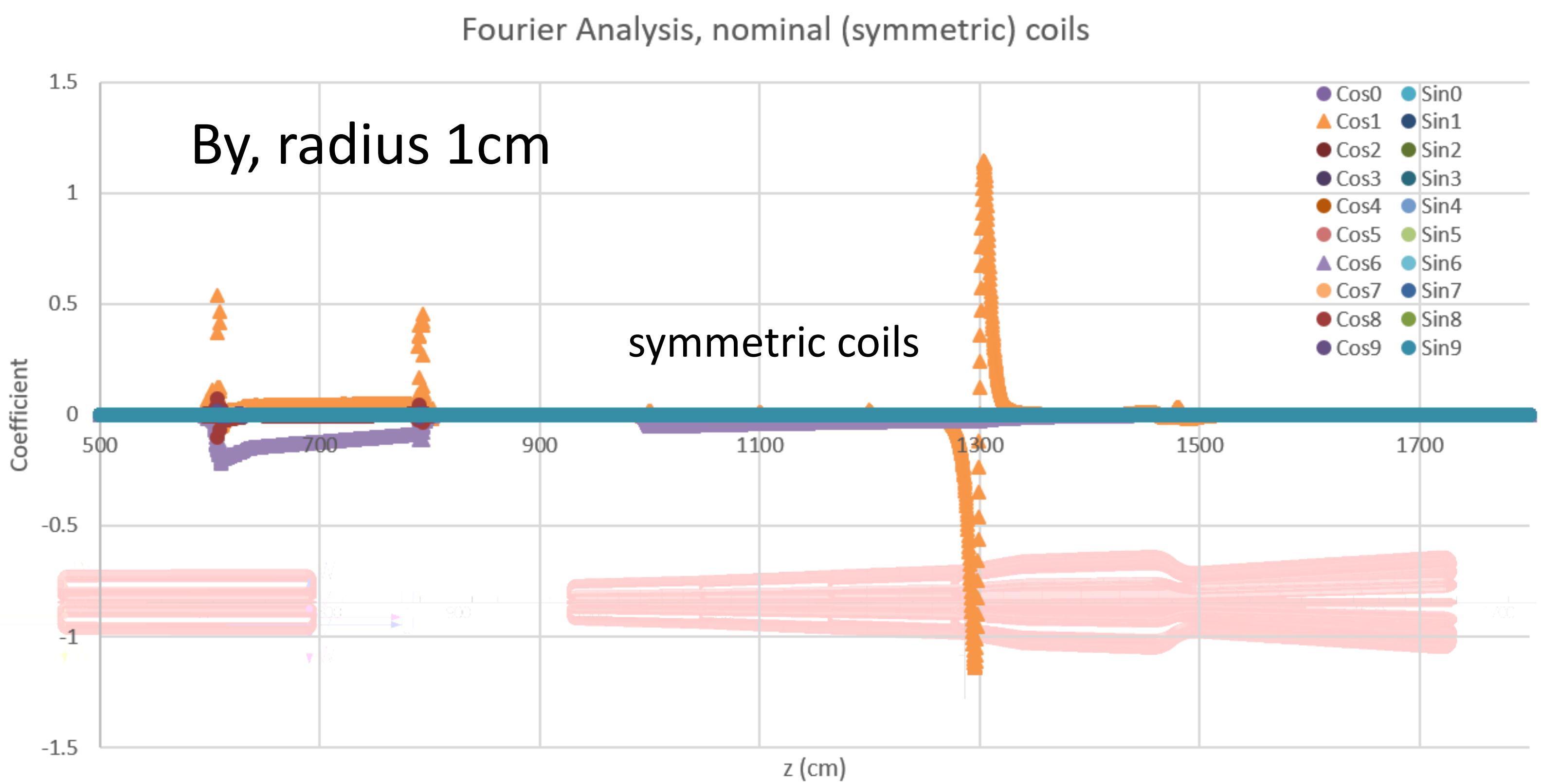
r = 4.08 cm

3mm outward – deflect left

Max vector shown: 150 G



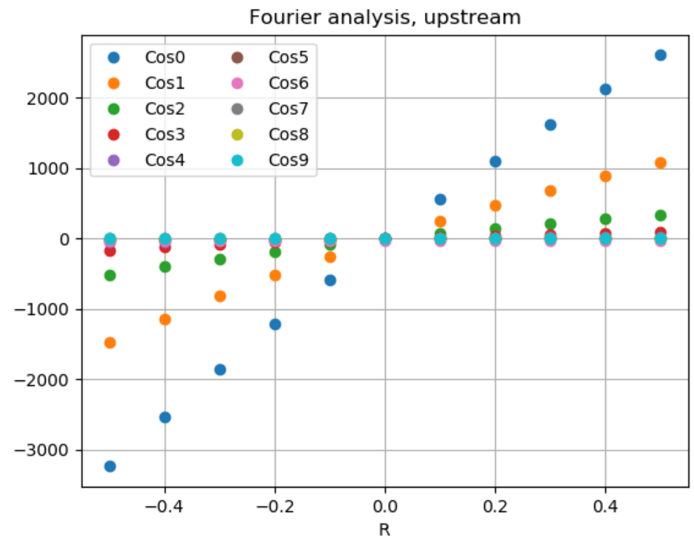
Fourier Decomposition

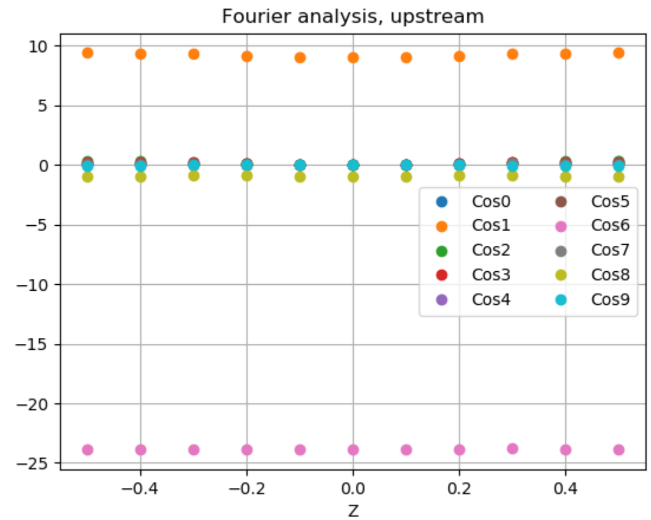


Fourier Decomposition

By, radius 1cm

Slope of Integral Gradients vs.Offset



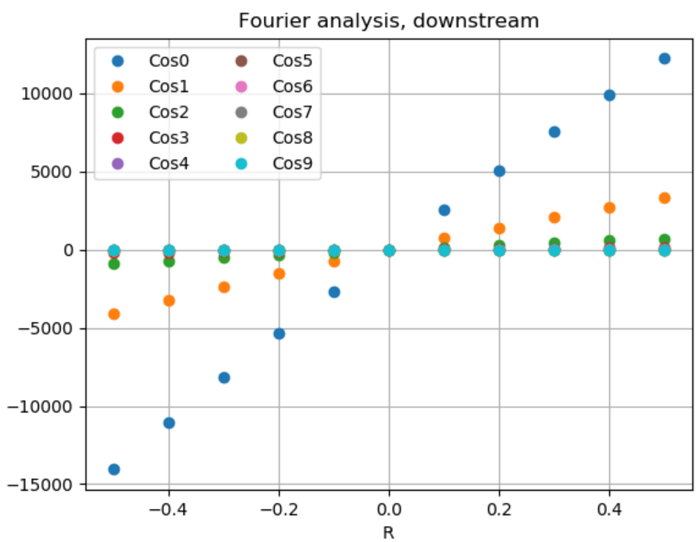


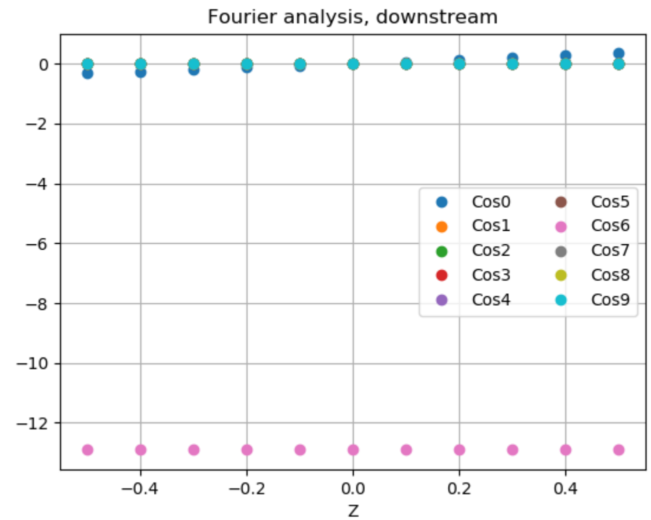
Upstream

R

Z

Downstream

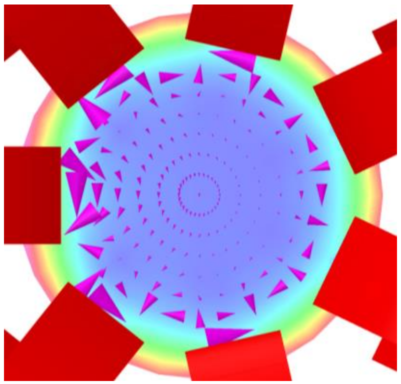




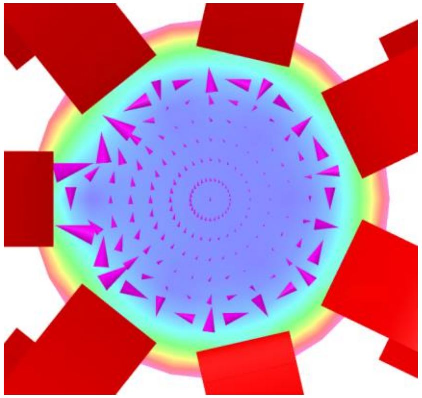
Effect on positrons

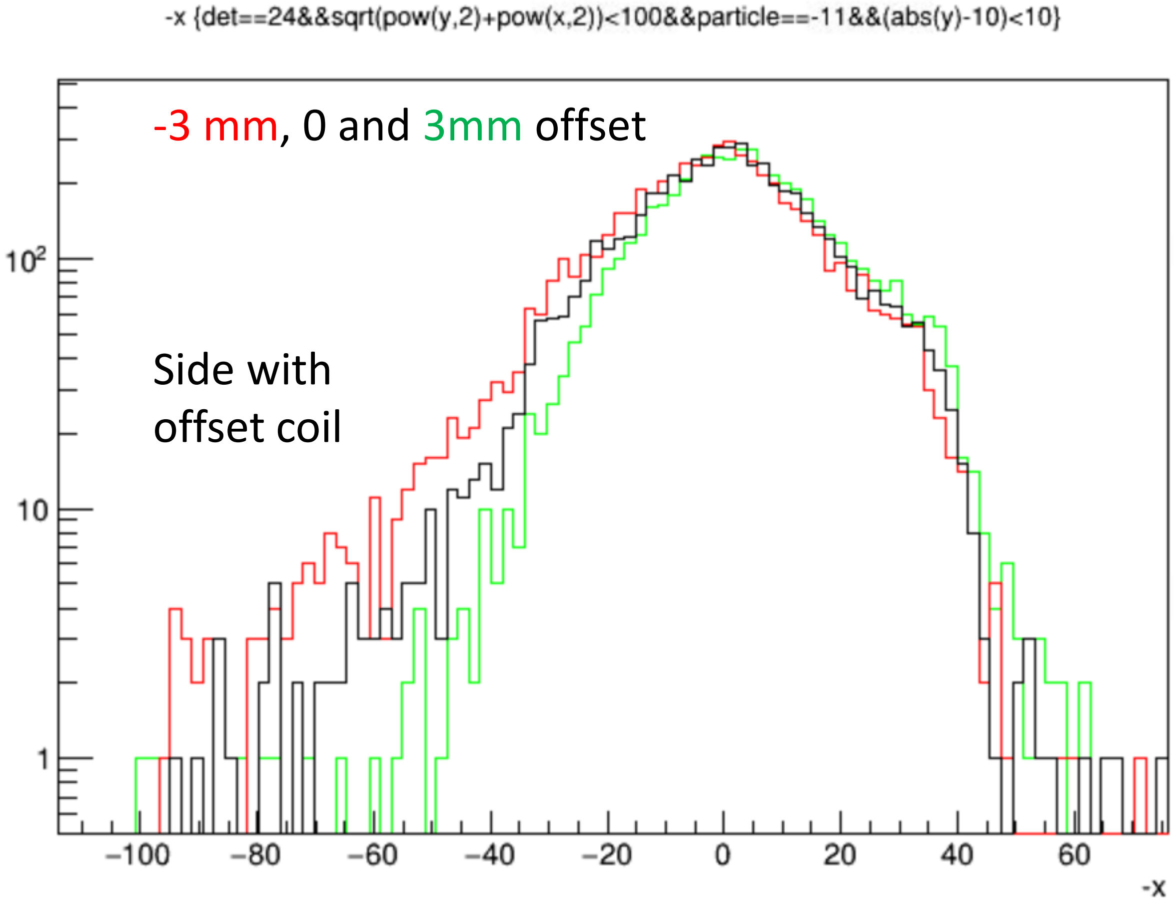
in horizontal band -2 < y <2 cm

-3mm (inward)

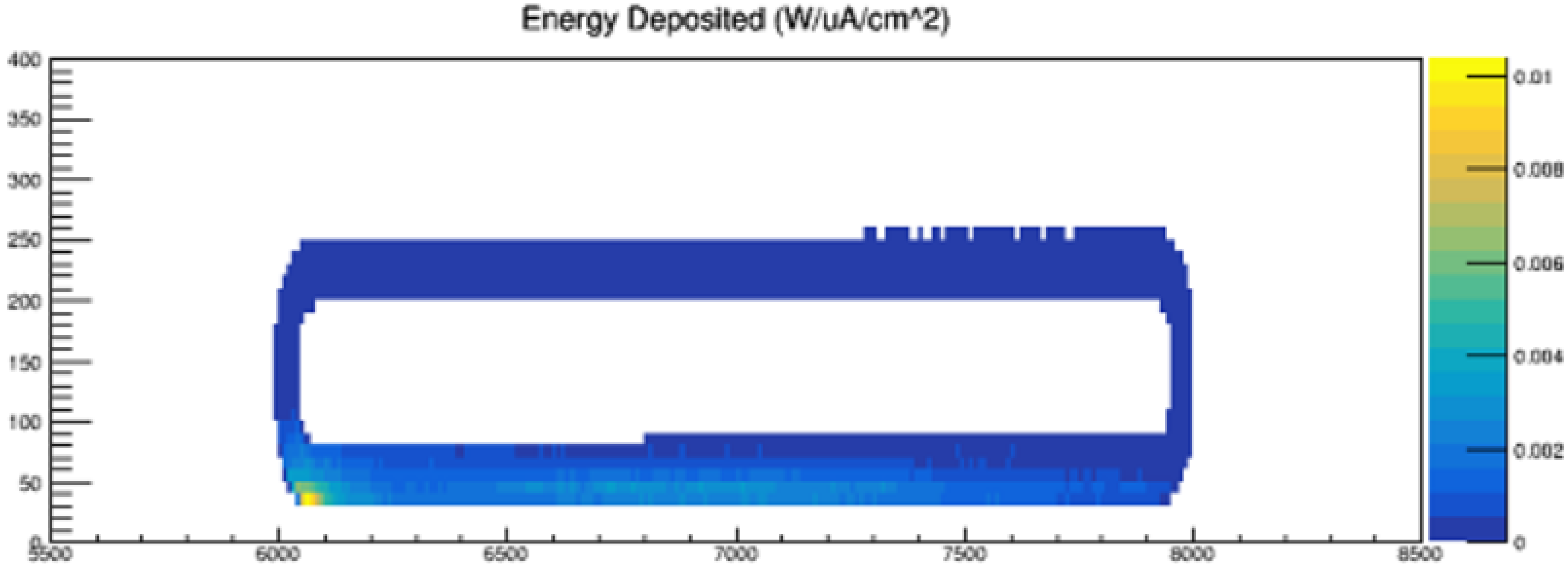


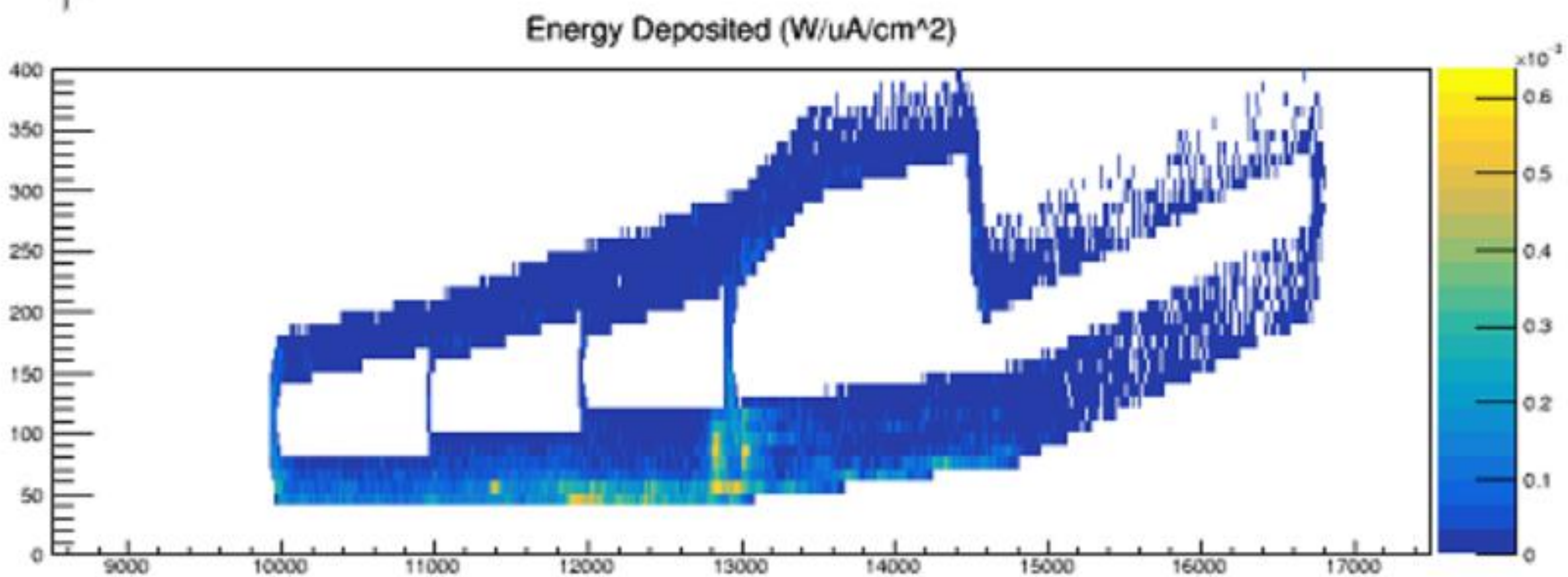
3mm (outward)



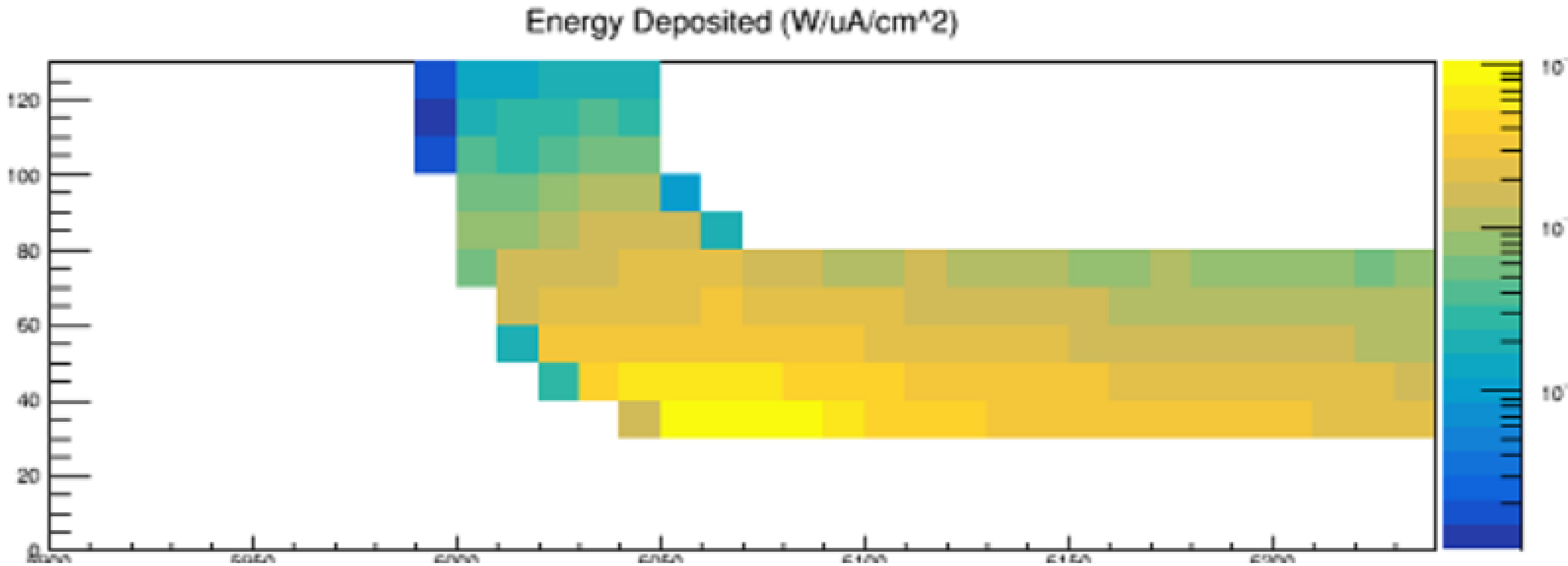


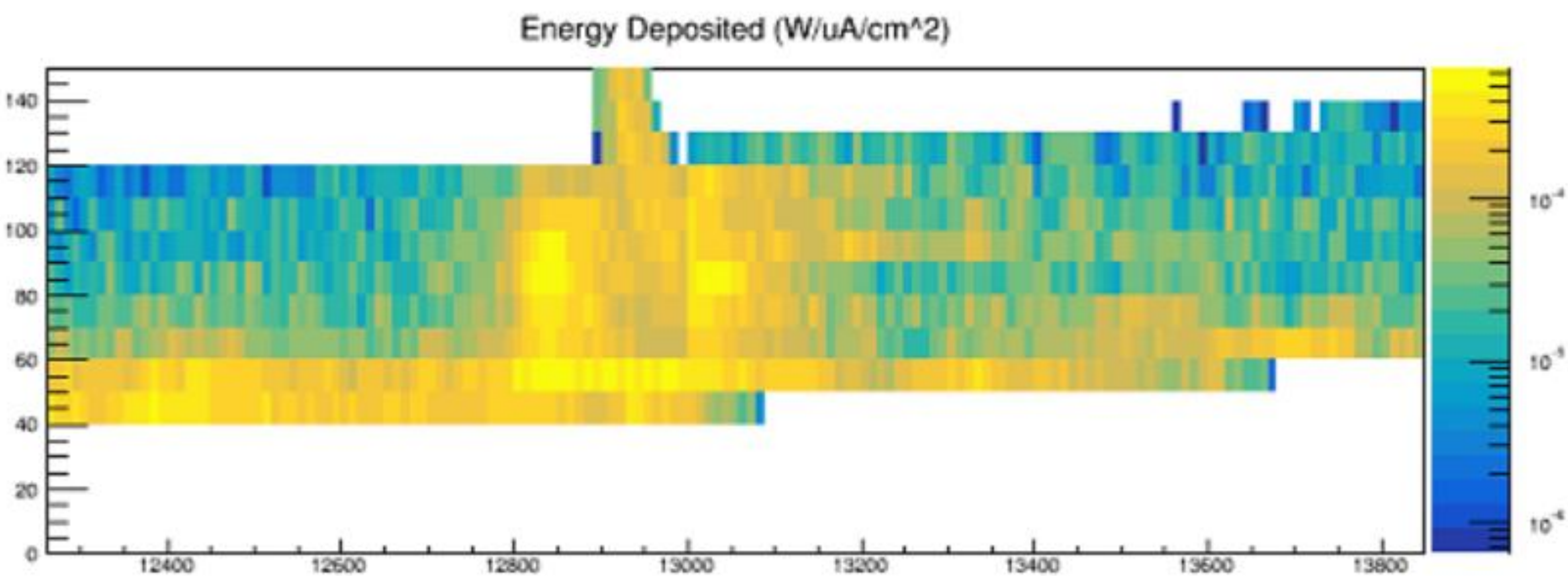
Energy Deposited on coils





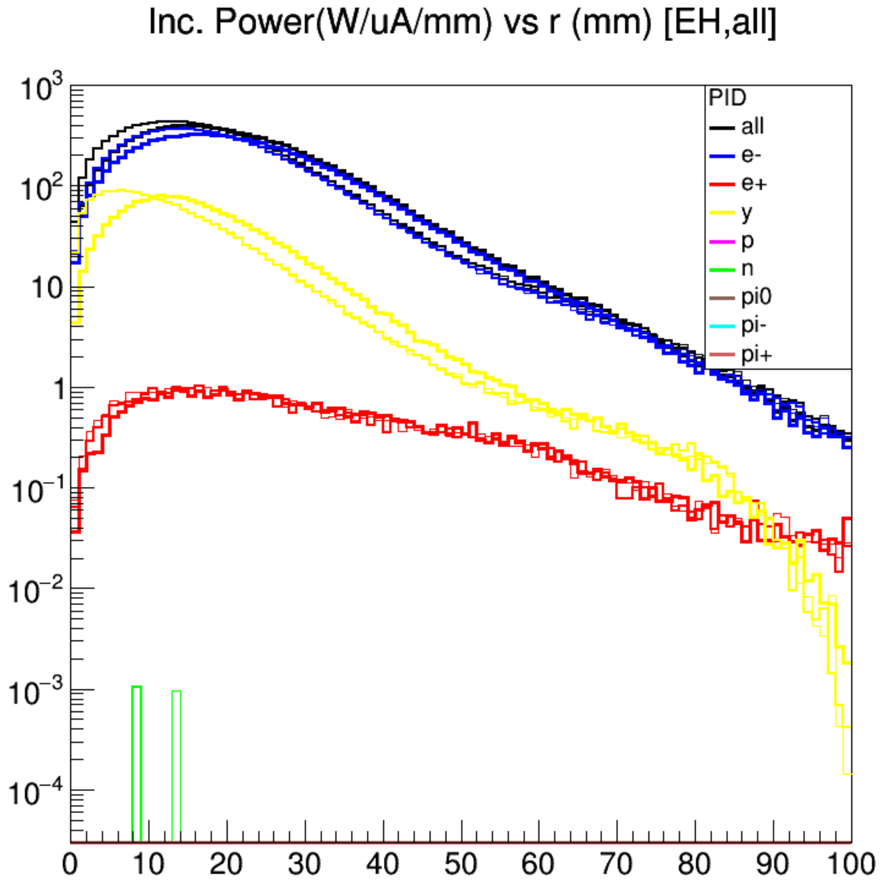
Doses on coils' hot spots

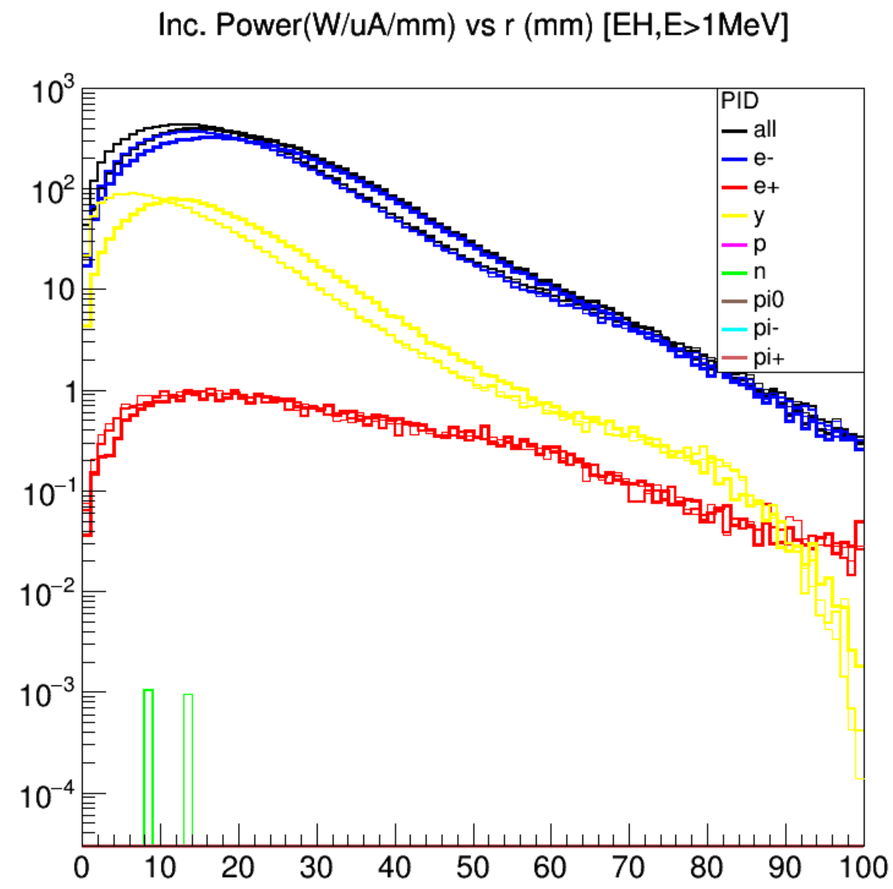




Preliminary Epoxy Dose Estimates

- Hot spot on upstream – 6.1×10^9 Rad
  - 86 mW/cm^2
  - Assuming 334 days @ 60 uA
- Rest of the upstream
  - 2×10^9 Rad
- downstream coils
  - 0.86 mW/cm^2
  - 6.1×10^7 Rad


Inc. Power(W/uA/mm) vs r (mm) [EH,all]
PID
all
e-
e+
y
p
n
pi0
pi-
pi+


Inc. Power(W/uA/mm) vs r (mm) [EH,E>1MeV]
PID
all
e-
e+
y
p
n
pi0
pi-
pi+

Installation, Commissioning and Tests

- Alignment
- Survey mechanically
- Field mapping (zero crossings)
- Tracking to "map" coils
- Find experimental axis
- Run with range of currents in coils (some higher?)
- Run with different beam energies
- Run with different raster sizes for target boiling studies

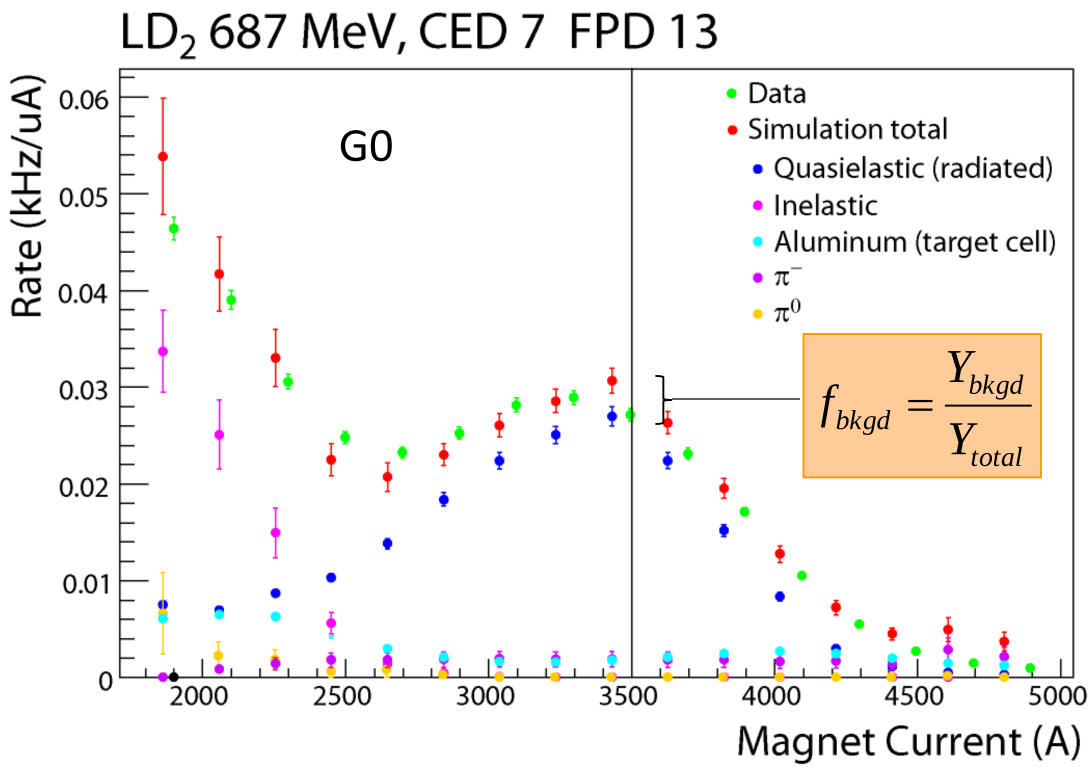
Background Corrections

$$A = \frac{A_{meas} - f_{bkgd} A_{bkgd}}{1 - f_{bkgd}}$$

$$f_{Al} \sim 10 - 15\%$$

$$f_{other} \sim 1\%$$

$$f_{\pi^-} \sim 5\%$$

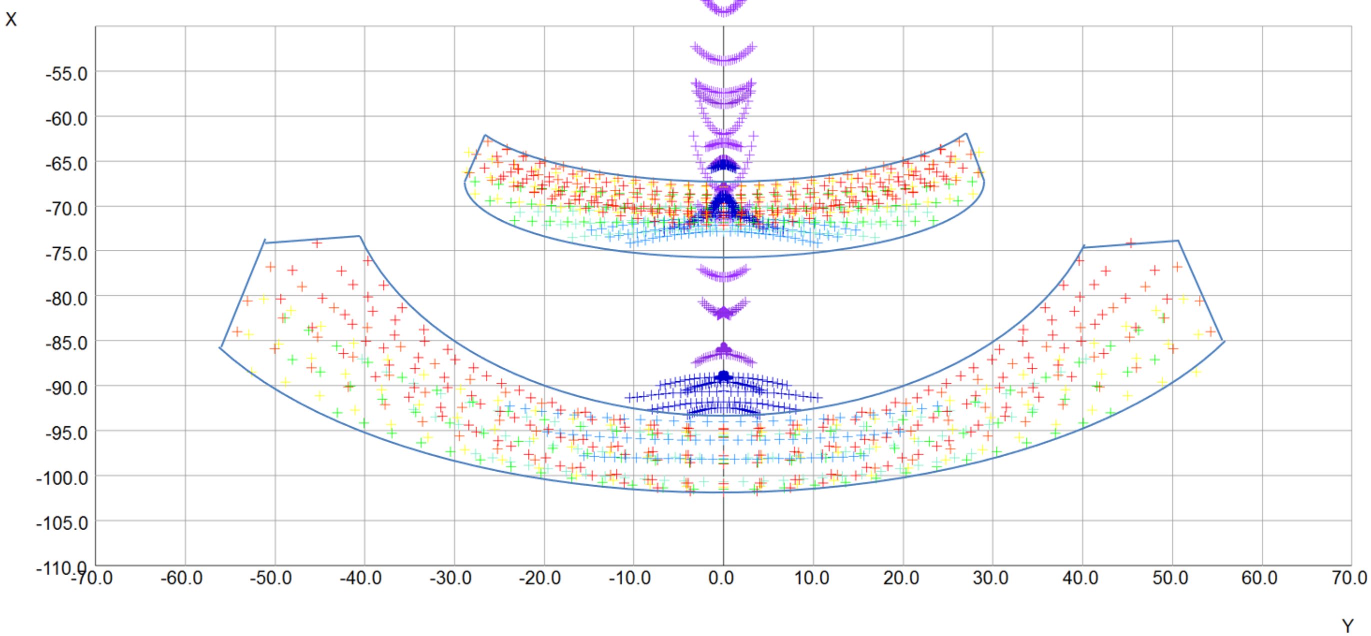


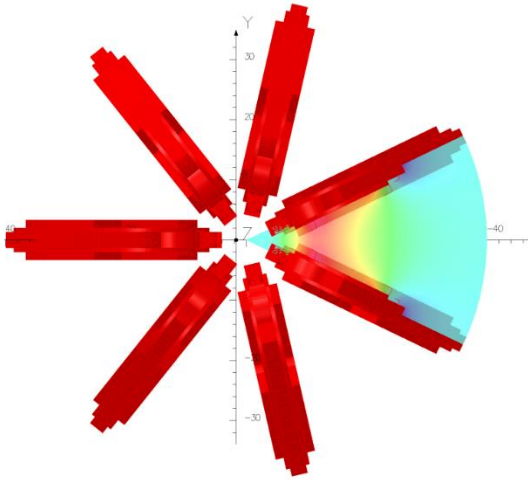
Alternatives

- Superconducting magnet
- Number of coils
- Iron in coils

- Different conductor cross-sections
- Segmented coils
- He vs. Air
- Smaller collimator openings

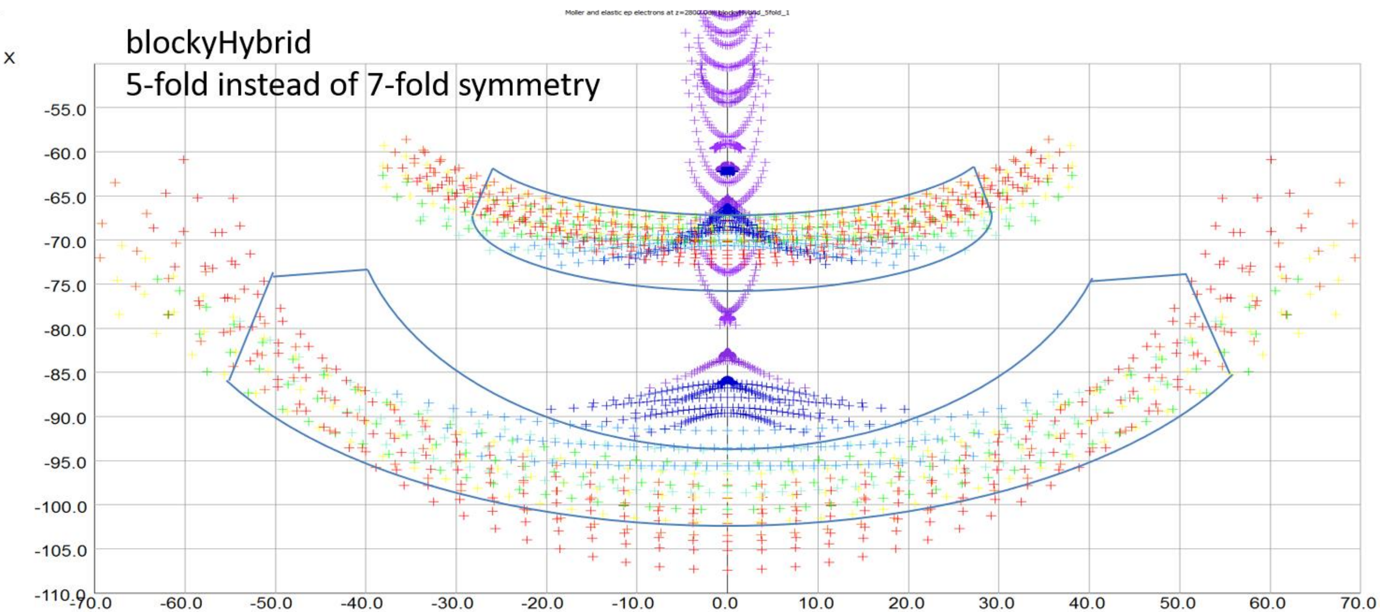
blockyHybrid

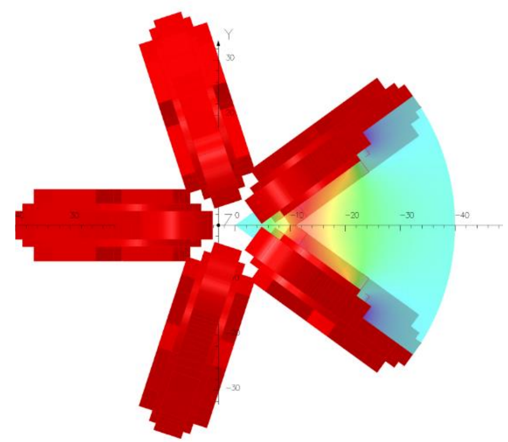




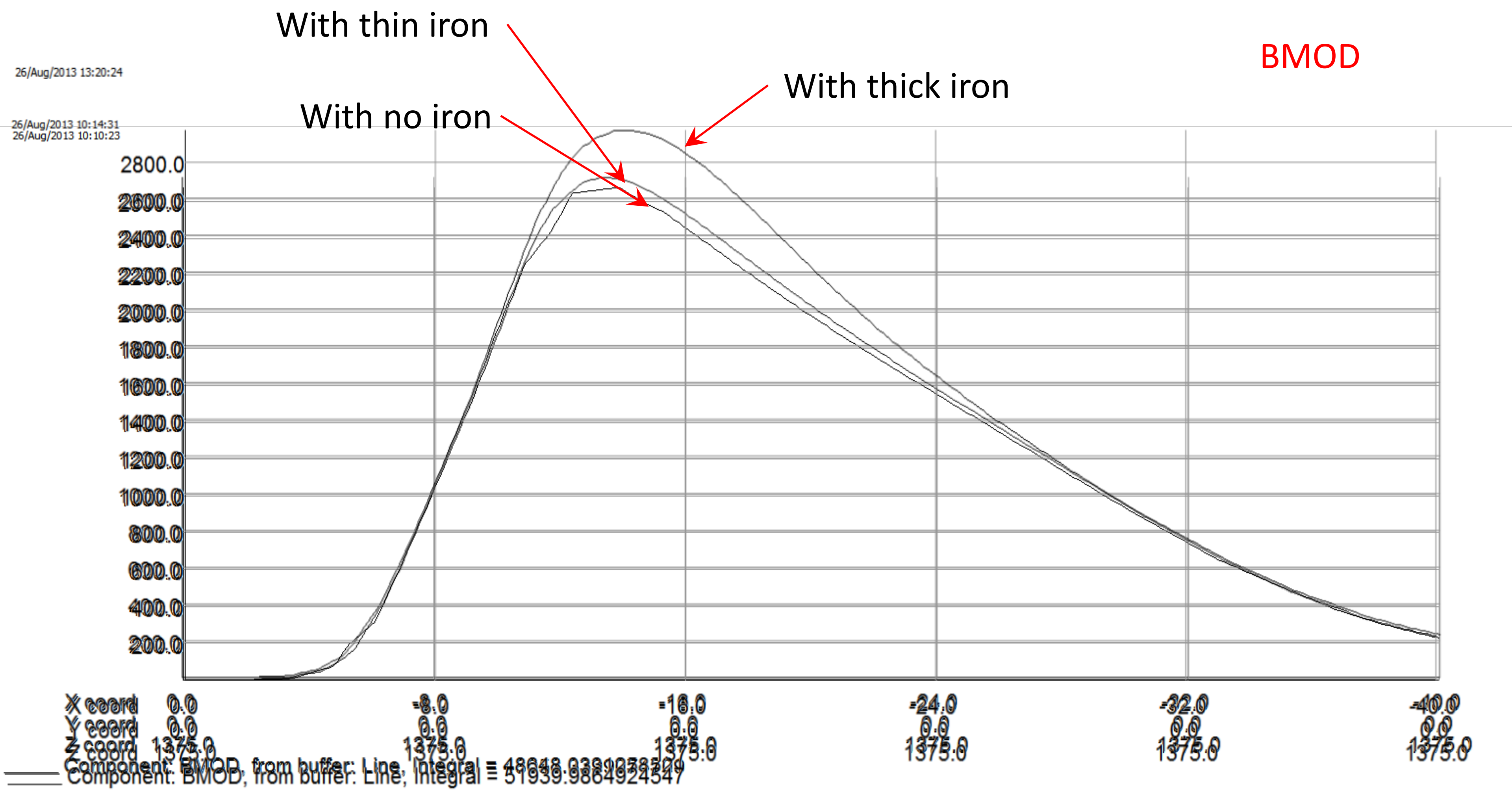
blockyHybrid

5-fold instead of 7-fold symmetry





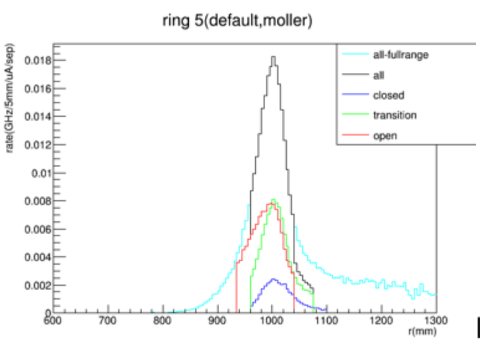
Iron in coils

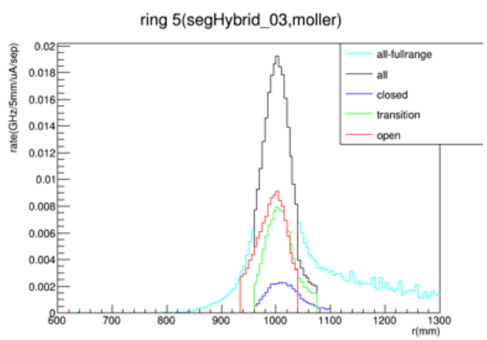


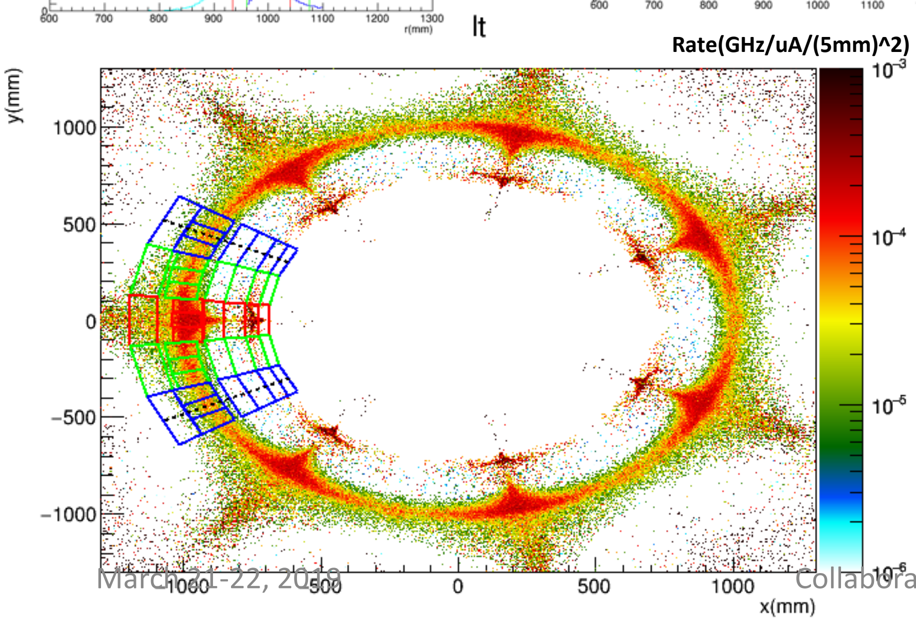
Comparison to Segmented

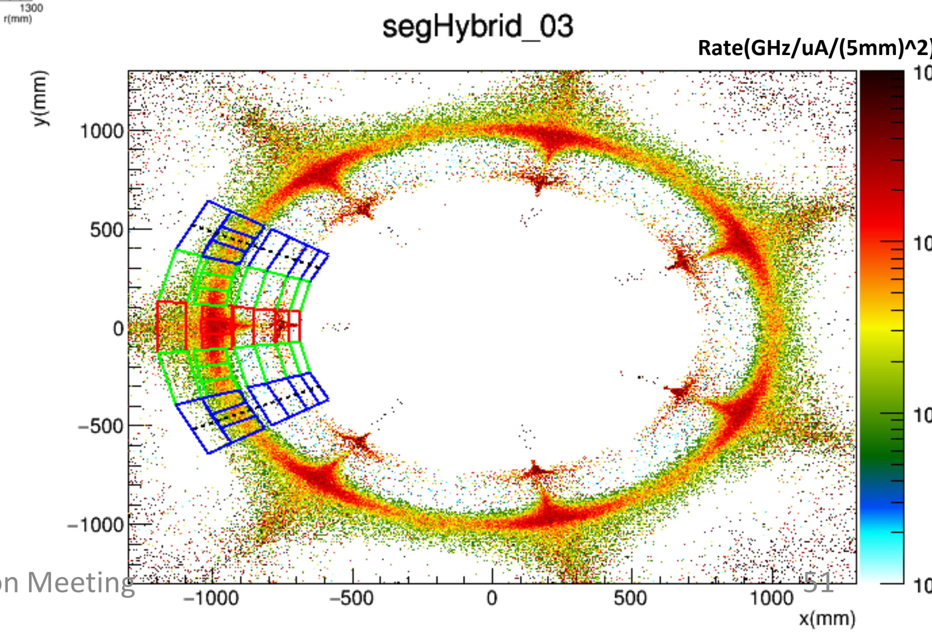
- segHybrid_03 produces almost identical distribution to default.
- The default sector definitions are shown in figures. Open in red, transition in green and closed in blue.

Sector	Generator	Rate	Fractional Rate
	inleastic	0.343	0.002
	elastic	20.087	0.118
default	moller	150.309	0.880
	inleastic	0.423	0.002
	elastic	20.488	0.116
segmented	moller	155.798	0.882



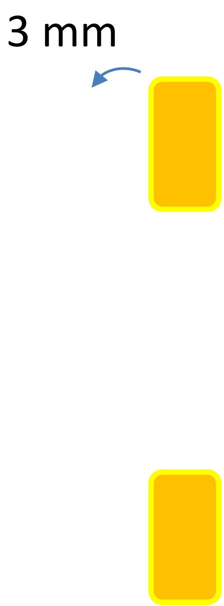


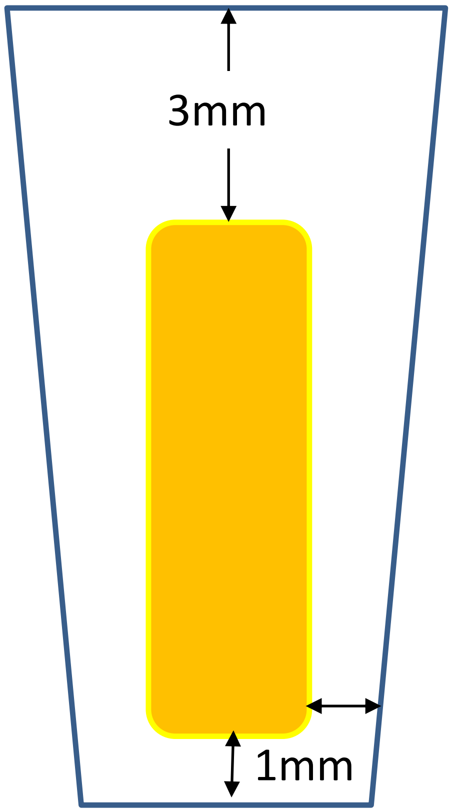




Defining Tolerances

- Coil envelopes
  - More sensitive to inner radius
- Particle envelopes (previous slides)
- Summary table (see next slide)



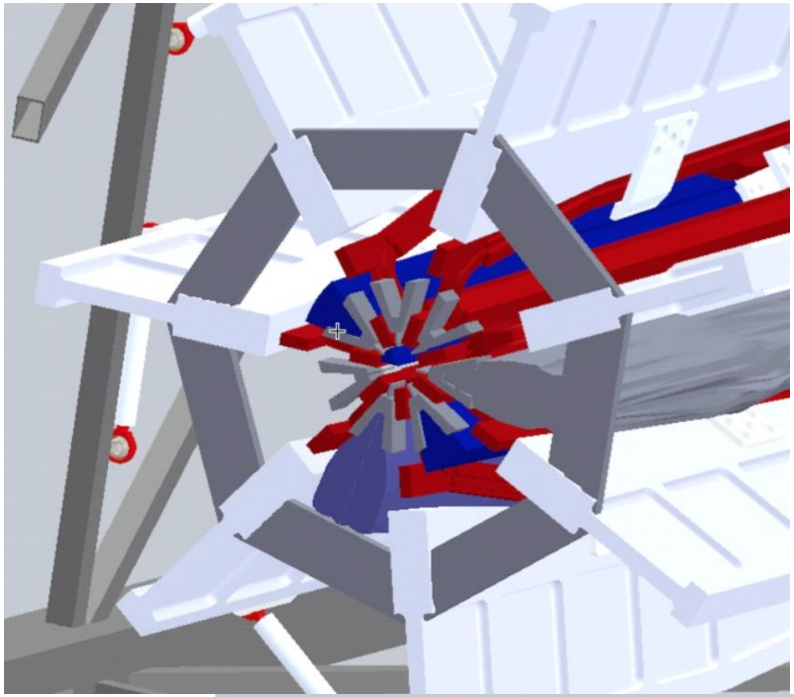


	Physics-driven Requirements	Value or Range		Comments
		Downstream Torus	Upstream Torus	
1	**Envelope for coils, strong backs and supports, relative to the beam line**	Z = ±25 mm R= +3 mm / -1 mm ϕ = 3mm outer radius, 1 mm inner radius	Same as hybrid	Provides limits for fabrication tolerances, assembly tolerances and movement during operation
2	**Material thickness budget in beam or particle path**	Nothing in primary beam; Al windows, possibly He	Same as hybrid	
3	**Level of vacuum for magnet chambers**	1 Torr	1 Torr	
4	**Expected total maximum radiation levels**	10^8 Rads	10^8 Rads	Dose in epoxy, not on copper/water
5	**Location accuracy of collimator, magnet and beam line centers relative to one another**	± 1mm	± 1mm	
6	**Magnetic field temporal stability**	Power stability = less than 25 ppm over 8 hours ; less than 50 ppm over 24 hours		
7	**Ampere-Turns per coil per magnet**	Zone A = 7752 (hybrid) ; Sub-Coil #1 = 8915 (seg) Zone B = 10602 (hybrid) ; Sub-Coil #2 = 12192 (seg) Zone C = 16862 (hybrid) ; Sub-Coil #3 = 19391 (seg) Zone D = 29160 (hybrid) ; Sub-Coil #4 = 33534 (seg)	4286	DS Torus: Segmented design has 15% more AT. Sub-Coil #1 is upstream-most coil
8	**Coils must be no closer than 5X the multiple scattering radius to beam center**	40 mm upstream, 50 mm middle	30 mm	To minimize radiation on the coils
9	**Collimator inner and outer diameter machining accuracy**	Collimator #2 = ± 200 μm		Coll 2 – acceptance Coll 1 – power deposition
	Collimator inner and outer diameter position accuracy	± 1 mm		
10	**Clearance between coils and particle envelopes**	1mm clearance at upstream ends, inner radius and 3mm everywhere else		

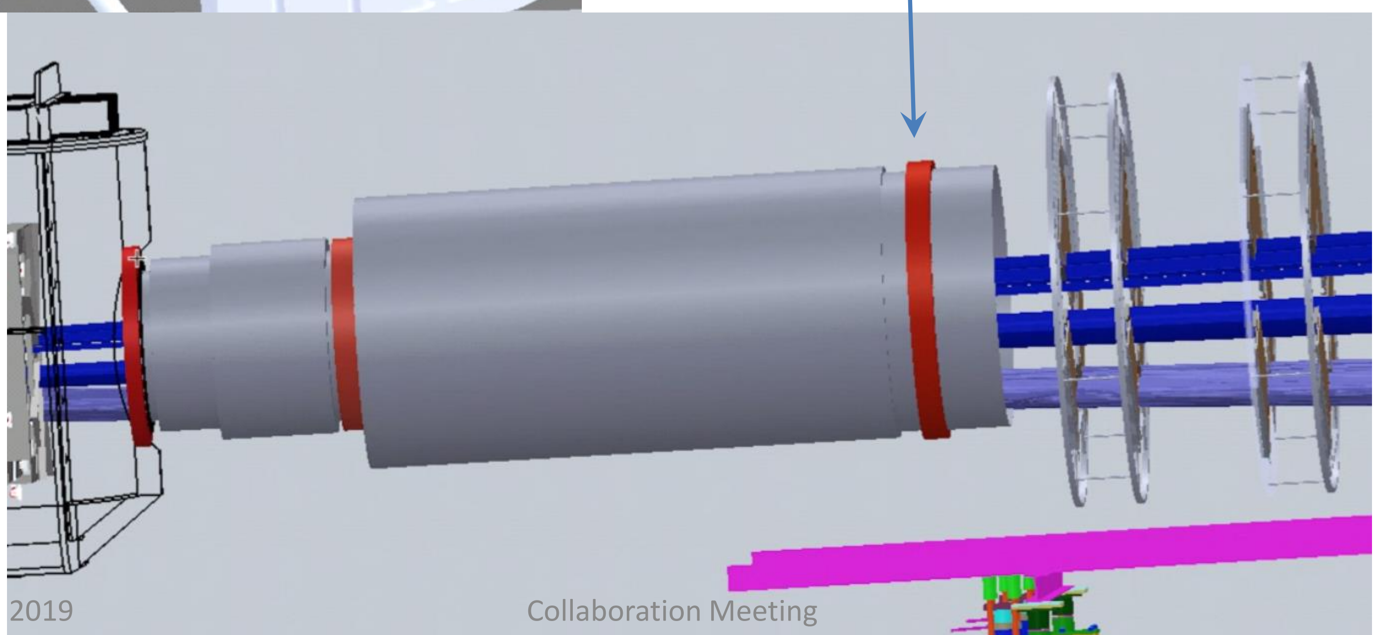
Further Studies

- Sensitivities
  - Finalize sensitivity, dose studies in report
  - Sensitivity to backgrounds in detectors
  - Check particle transport to dump
  - Power in collimator 1
- Simulate effect of solenoidal field from power connections
- Model epoxy in GEANT4
- Design beampipe

Backup slides

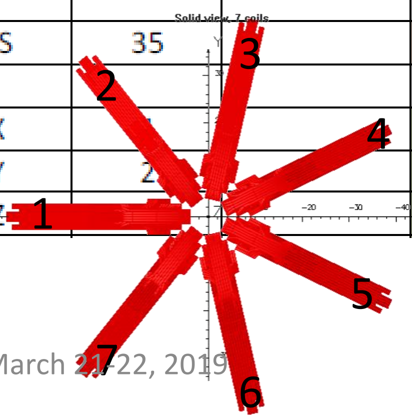


Ring 7 will have quartz detectors after it



Forces

New CURD2 = 0 Force on horizontal coil (lbs)					
Segment	Cond. #	Length (cm)	F_x	F_y	F_z
A	4, 5, 6		-1575.89	-427.59	27.04
B	7, 8, 9		-1120.77	-122.59	69.43
C	10, 11, 12		-0.96	-5.33	65.79
D	13, 14, 15		912.52	-72.03	-58.37
E	16, 17, 18		87.05	-13.57	31.72
F	19, 20, 21		60.35	-12.61	23.20
G	22, 23, 24		370.23	-82.52	-12.93
H	25, 26, 27		200.05	-42.92	-34.45
IJ	28, 29		-13.54	-11.31	-84.93
K	30		153.81	-29.25	-6.51
LM	31		-1.07	-0.75	-11.76
N	32		87.13	-16.77	-3.69
O	33		-0.60	-0.42	-6.03
Q	34		51.79	-10.17	-1.51
RS	35		-1.43	-2.19	-8.24
X	1		-106.67	-28.75	0.27
Y	2		-181.23	-48.18	0.47
Z	[illegible]		-319.04	-83.80	0.82



Old

	F_x (lbs)	F_y (lbs)	F_z (lbs)
Total	-2941.75	0.00	36.88
Inner	-5434.03	-7.24E-08	1.05E+02
Outer	2636.429	-2.80E-08	-84.7468
returns	-144.153	1.71E-10	16.45164
	-2941.75	0.00	36.88

New

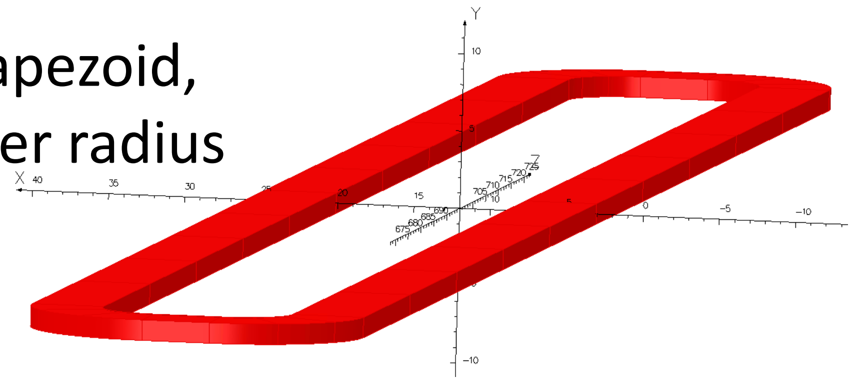
	F_x (lbs)	F_y (lbs)	F_z (lbs)
Total	-1883.92	0.00	-9.34
Inner	-4111.14	0.00	119.65
Outer	2013.15	0.00	-30.87
returns	214.07	0.00	-98.13
	-1883.92	0.00	-9.34

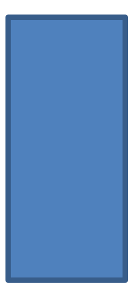
Coil off

	F_x (lbs)	F_y (lbs)	F_z (lbs)
Total	-1398.25	-1010.75	-9.66
Inner	-3303.59	-710.91	98.03
Outer	1722.88	-236.92	-28.08
returns	182.45	-62.92	-79.61
	-1398.25	-1010.75	-9.66

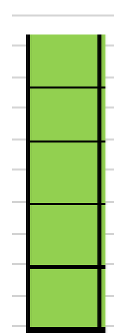
Comparisons between

- Default – trapezoidal blocky model
- Rectangle 1 – same height as trapezoid, same width as inner radius
- Rectangle 2 – Dave's version 10
- Rectangle 3 – Dave's version 13



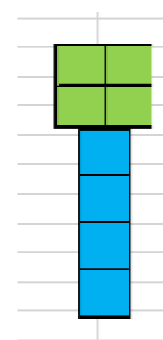


13.3 x 46.4 mm^2



10

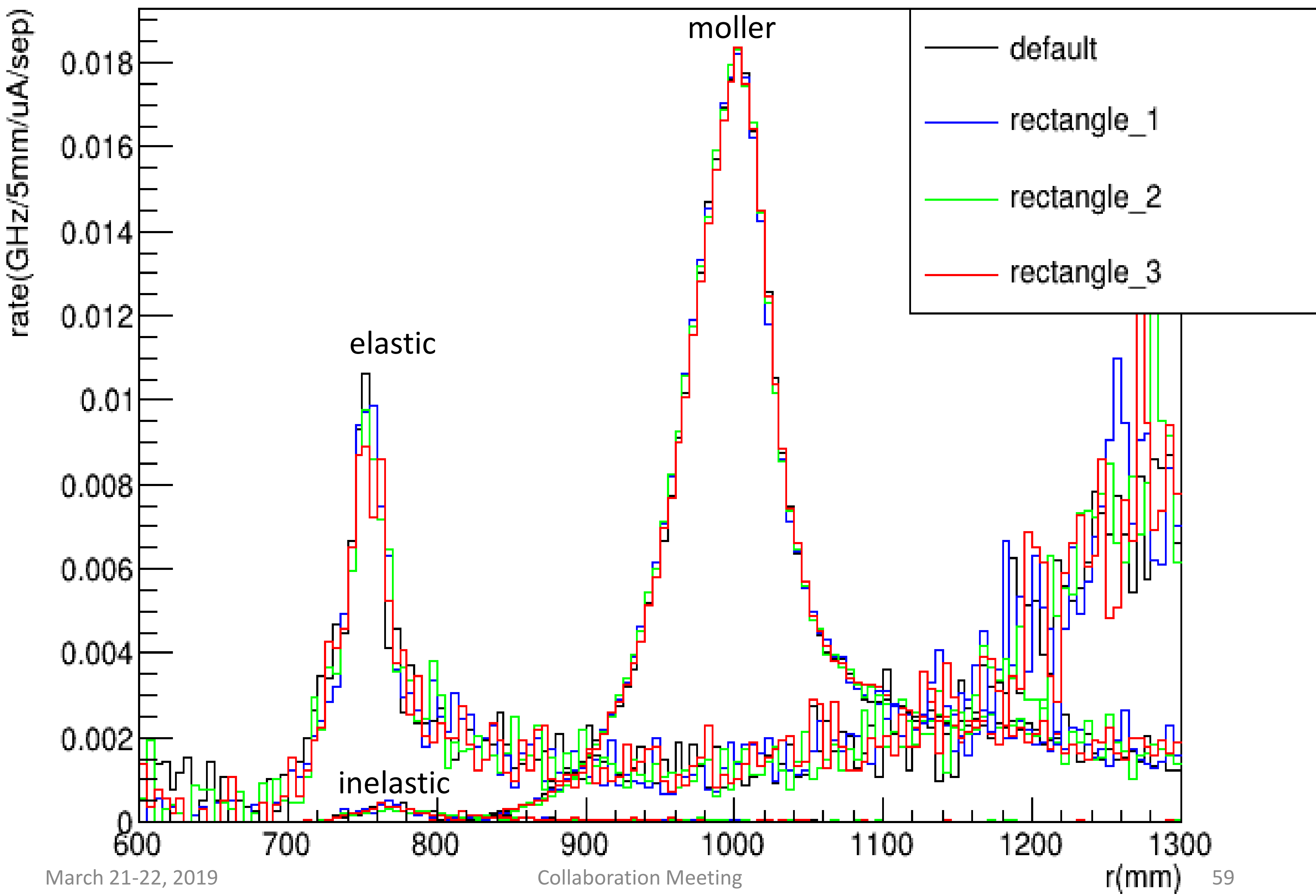
9.2x51.6 mm^2

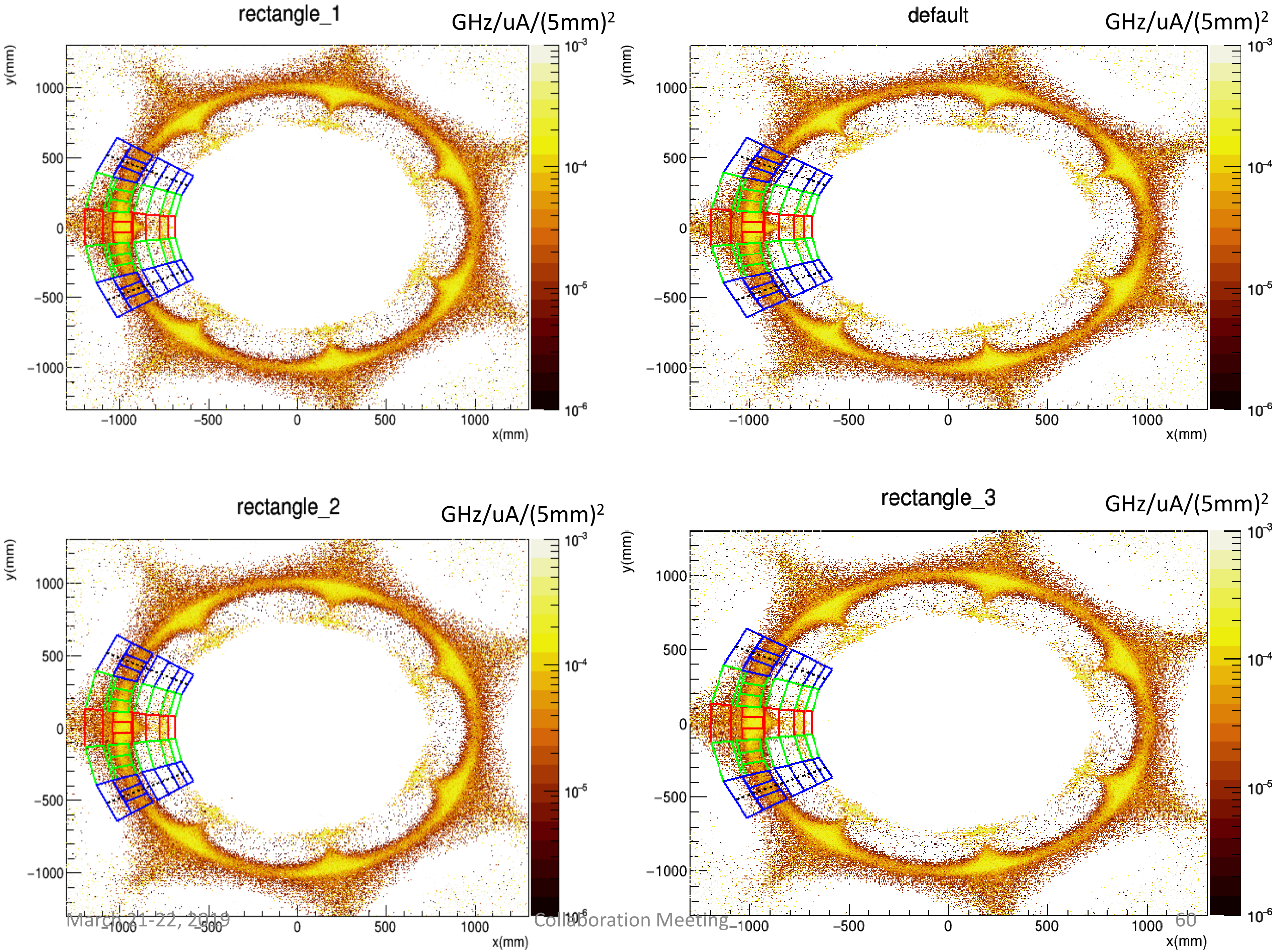


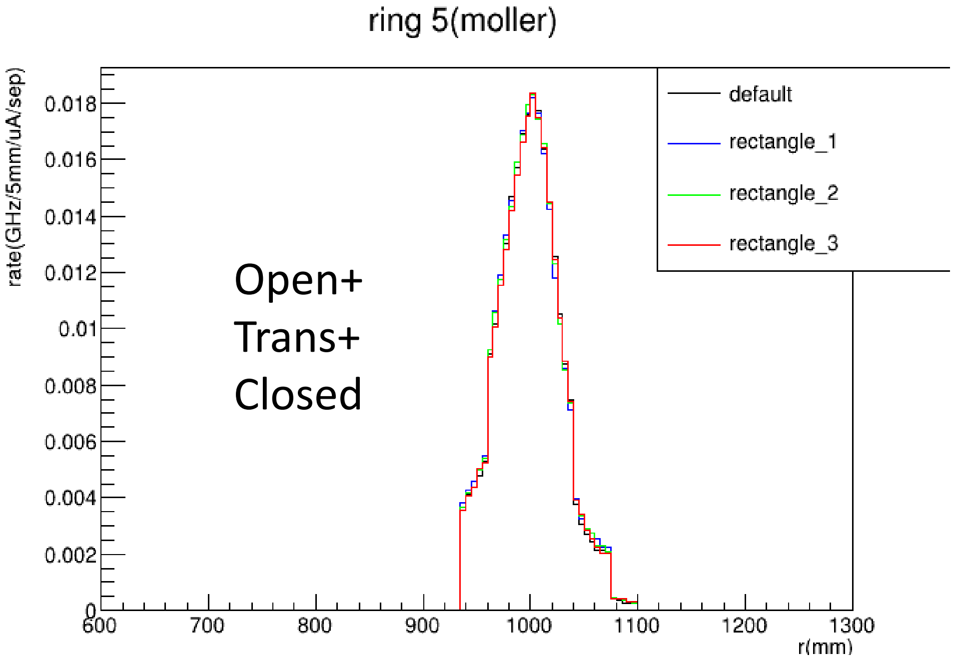
13

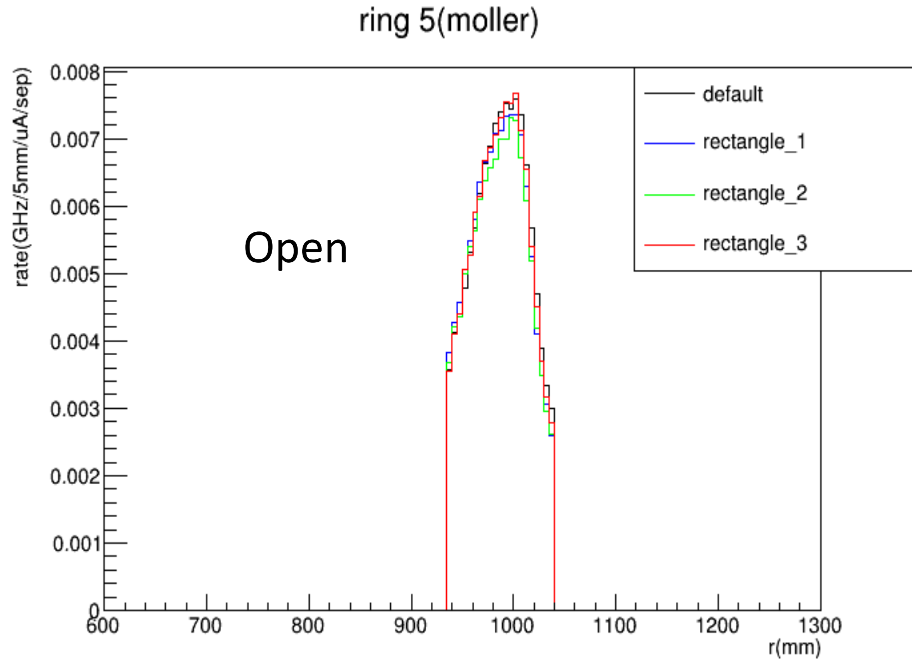
15.9 or 8.2 x 46.8 mm^2

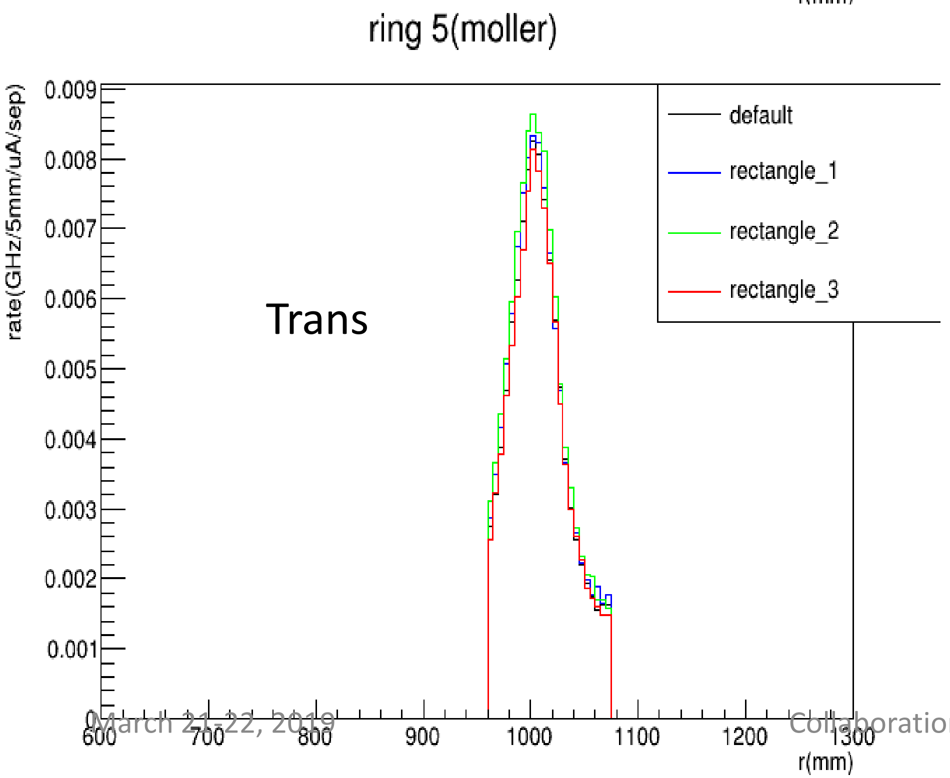
Overall Radial Distribution




rectangle_1
GHz/uA/(5mm)^2
default
GHz/uA/(5mm)^2
rectangle_2
GHz/uA/(5mm)^2
rectangle_3
GHz/uA/(5mm)^2
y(mm)
x(mm)
10^{-3}
10^{-4}
10^{-5}
10^{-6}


ring 5(moller)
rate(GHz/5mm/uA/sep)
r(mm)
default
rectangle_1
rectangle_2
rectangle_3
Open+
Trans+
Closed


ring 5(moller)
rate(GHz/5mm/uA/sep)
r(mm)
default
rectangle_1
rectangle_2
rectangle_3
Open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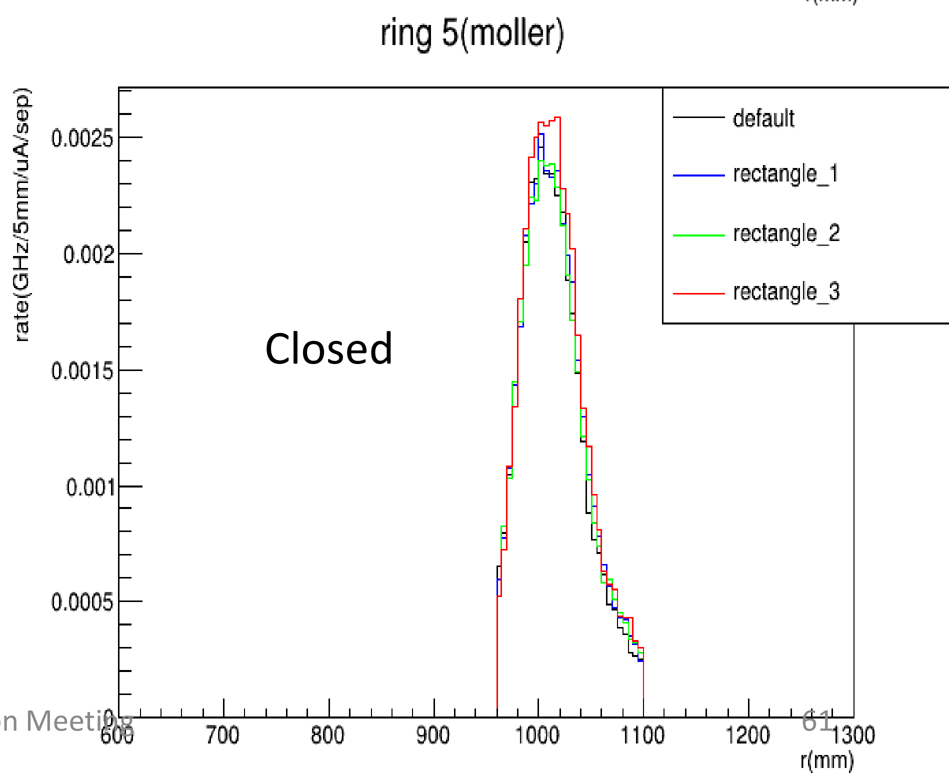
rate(GHz/5mm/uA/sep)
r(mm)
default
rectangle_1
rectangle_2
rectangle_3
Trans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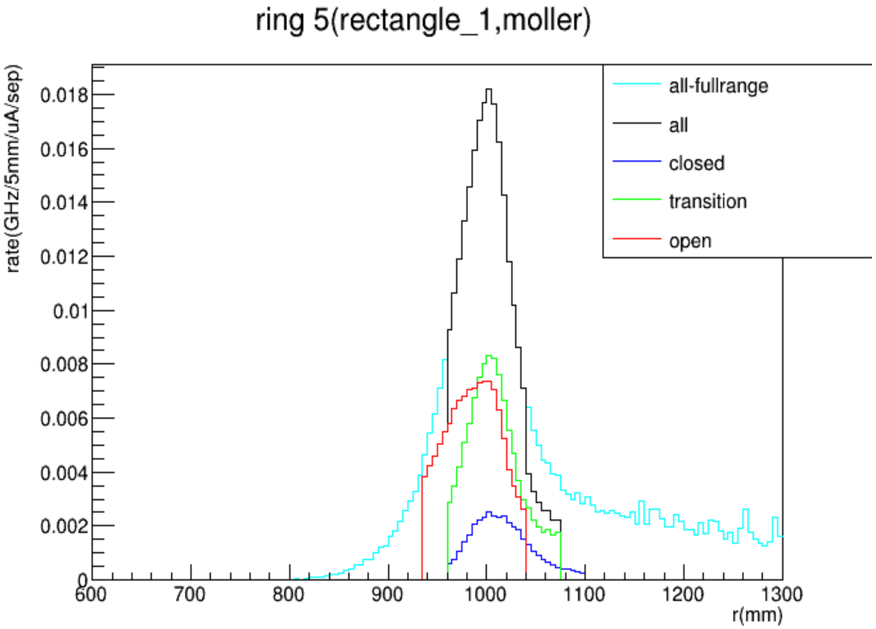
rate(GHz/5mm/uA/sep)
r(mm)
default
rectangle_1
rectangle_2
rectangle_3
Closed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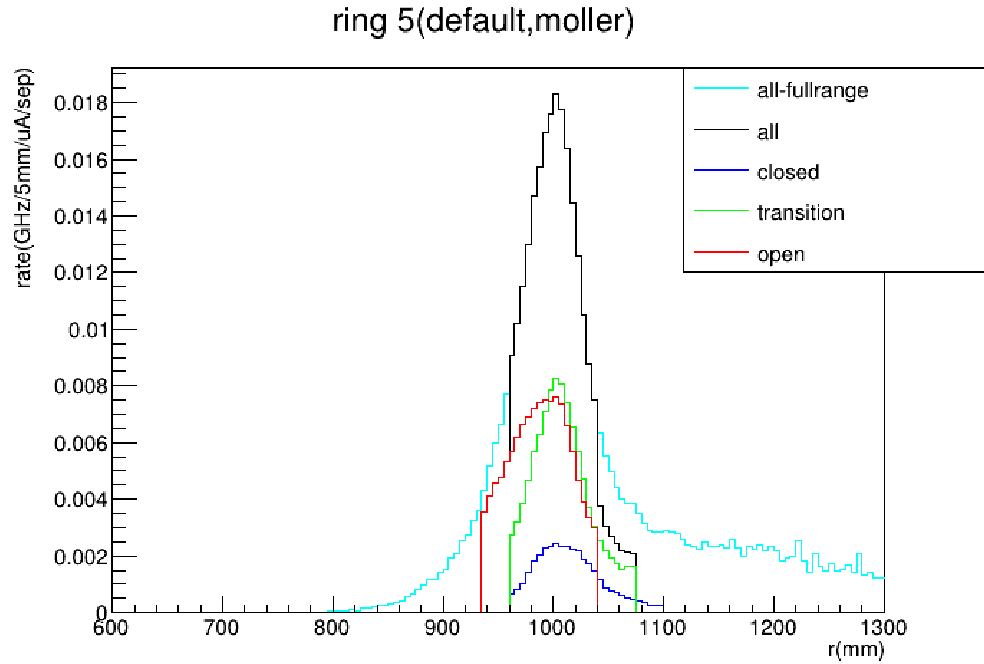
rate(GHz/5mm/uA/sep)
r(mm)
all-fullrange
all
closed
transition
open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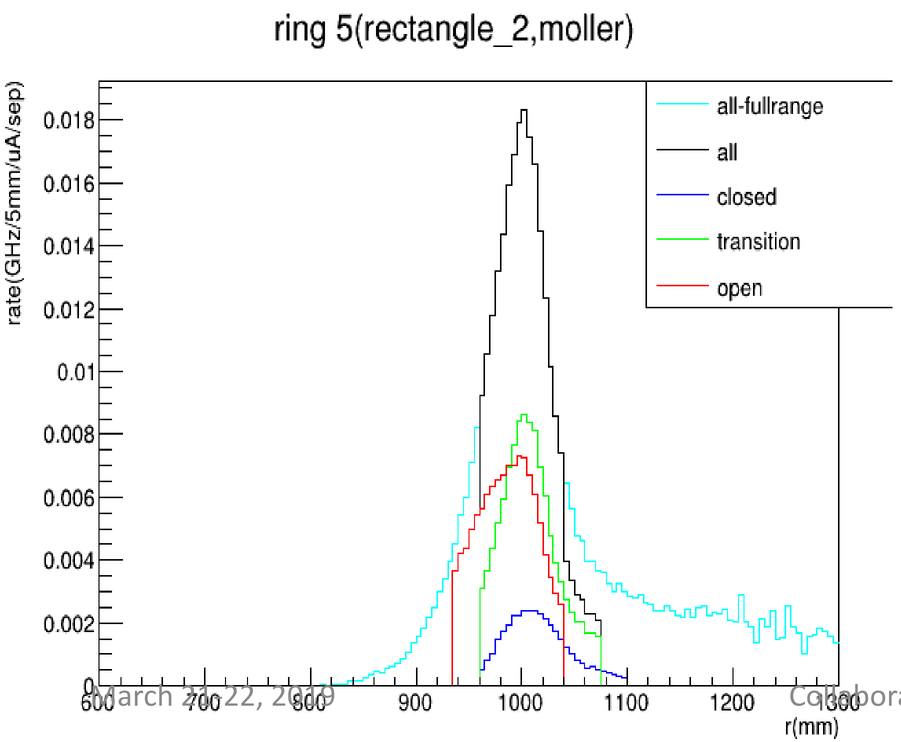
rate(GHz/5mm/uA/sep)
r(mm)
all-fullrange
all
closed
transition
open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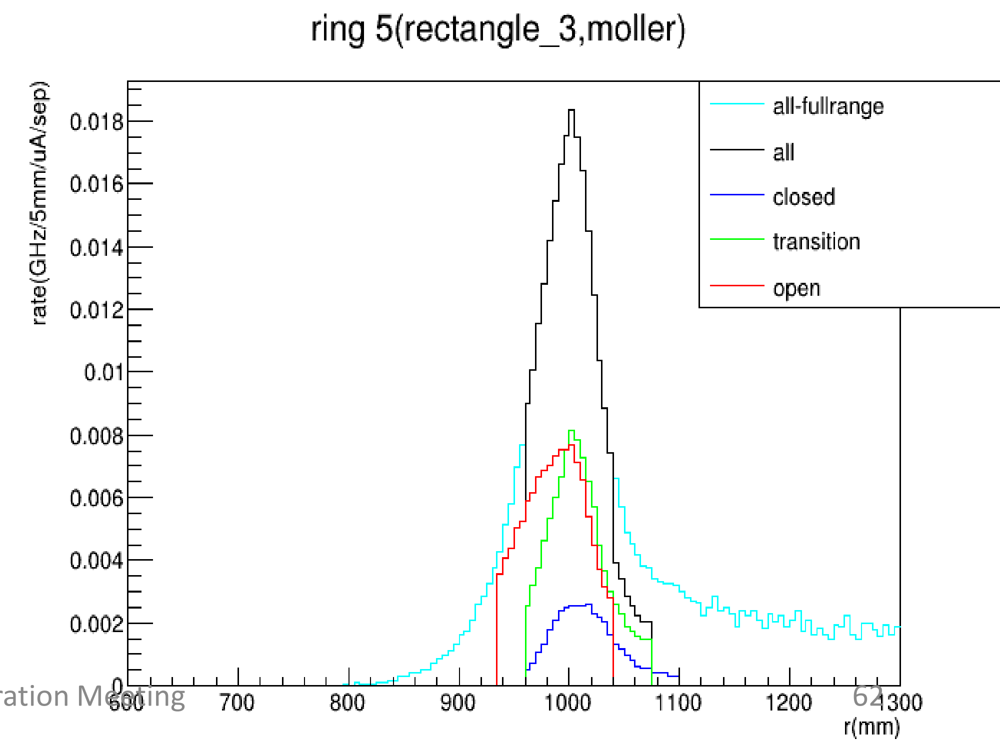
rate(GHz/5mm/uA/sep)
r(mm)
all-fullrange
all
closed
transition
open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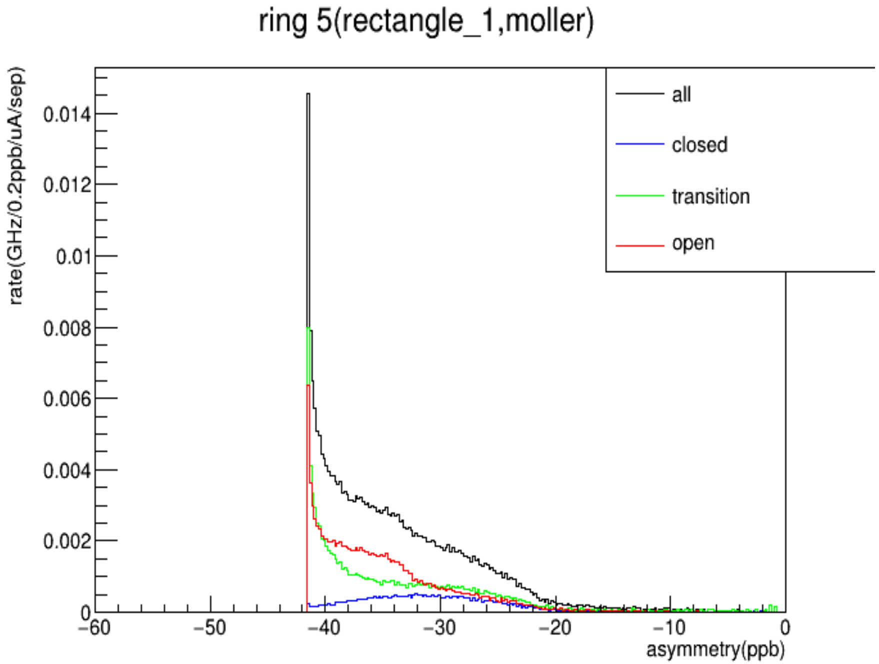
rate(GHz/5mm/uA/sep)
r(mm)
all-fullrange
all
closed
transition
open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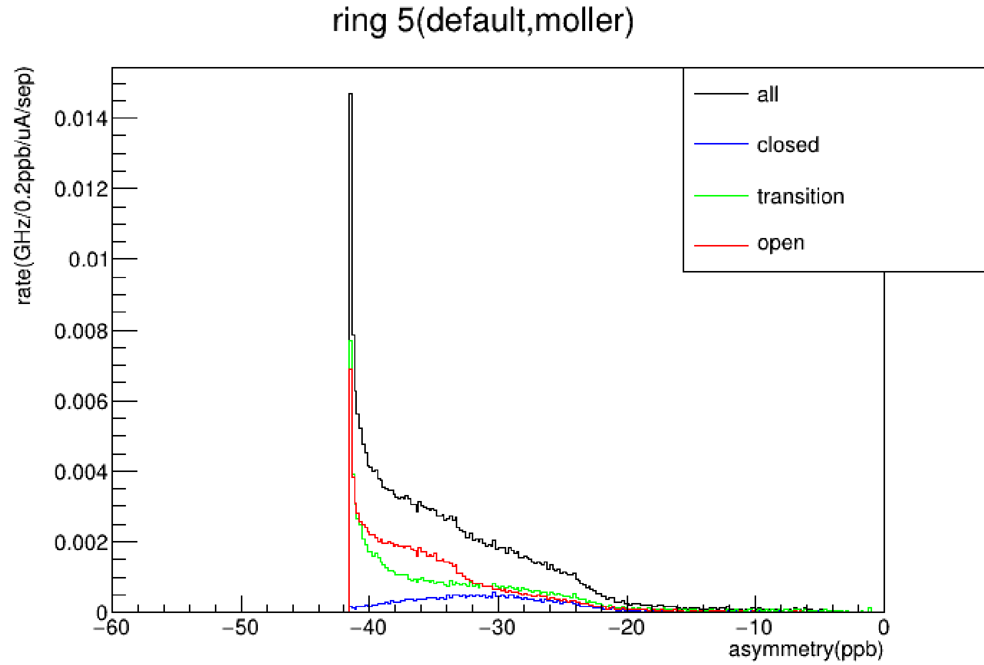
rate(GHz/0.2ppb/uA/sep)
asymmetry(ppb)
all
closed
transition
open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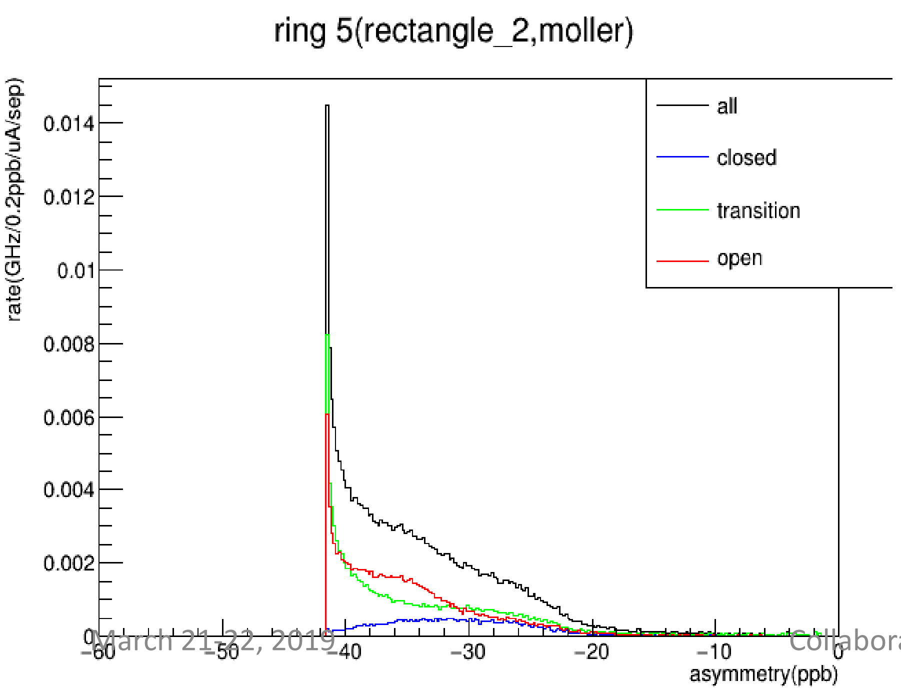
rate(GHz/0.2ppb/uA/sep)
asymmetry(ppb)
all
closed
transition
open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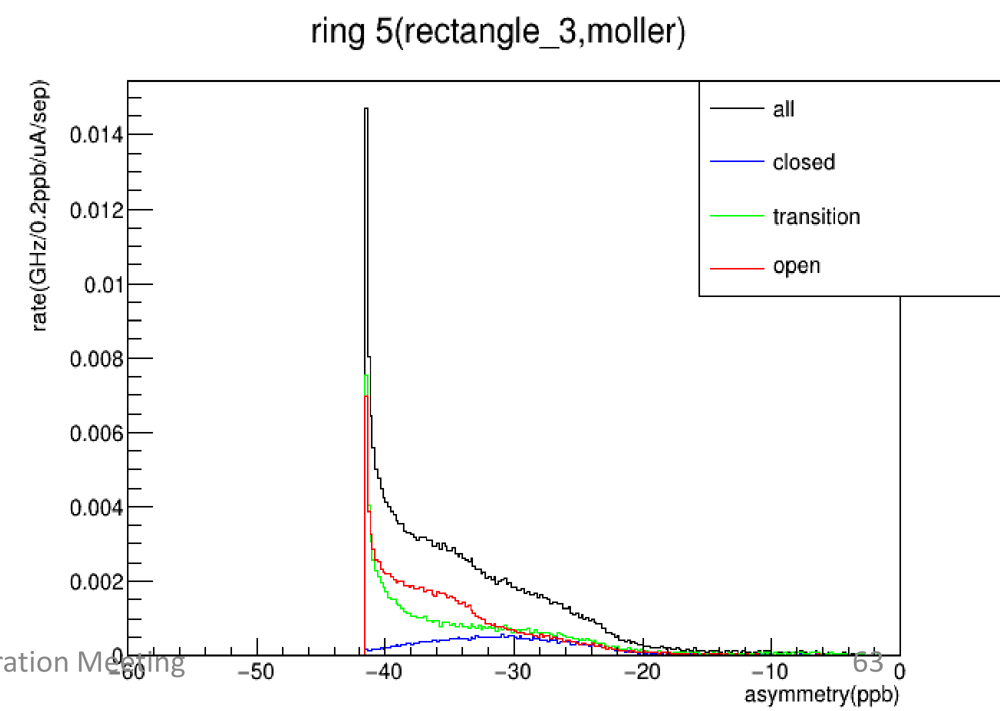
asymmetry(ppb)
all
closed
transition
open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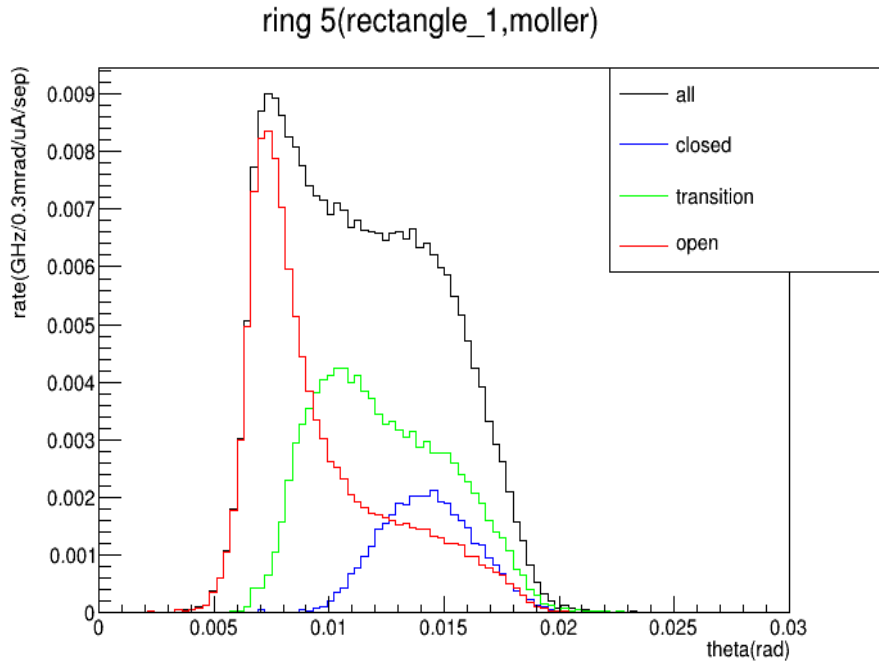
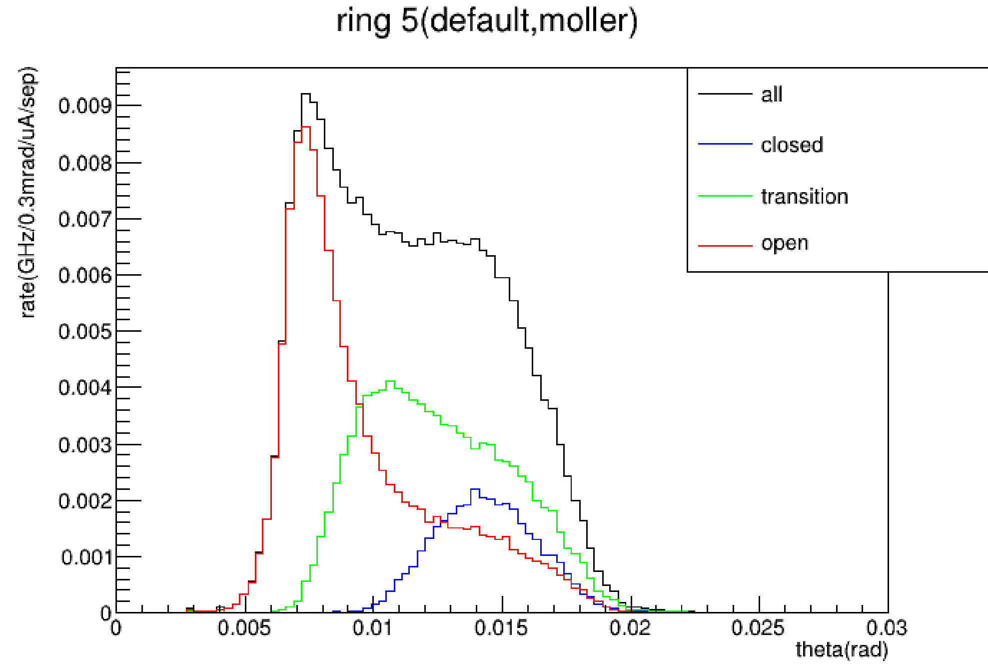
rate(GHz/0.2ppb/uA/sep)
asymmetry(ppb)
all
closed
transition
open


ring 5(rectangle_1,moller)
rate(GHz/0.3mrad/uA/sep)
theta(rad)
all
closed
transition
open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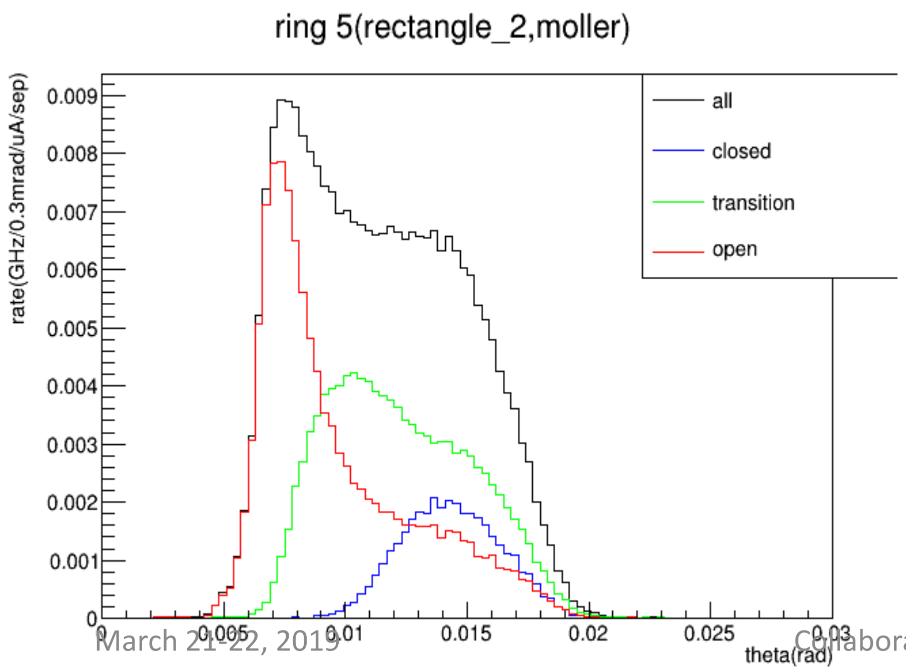
rate(GHz/0.3mrad/uA/sep)
theta(rad)
all
closed
transition
open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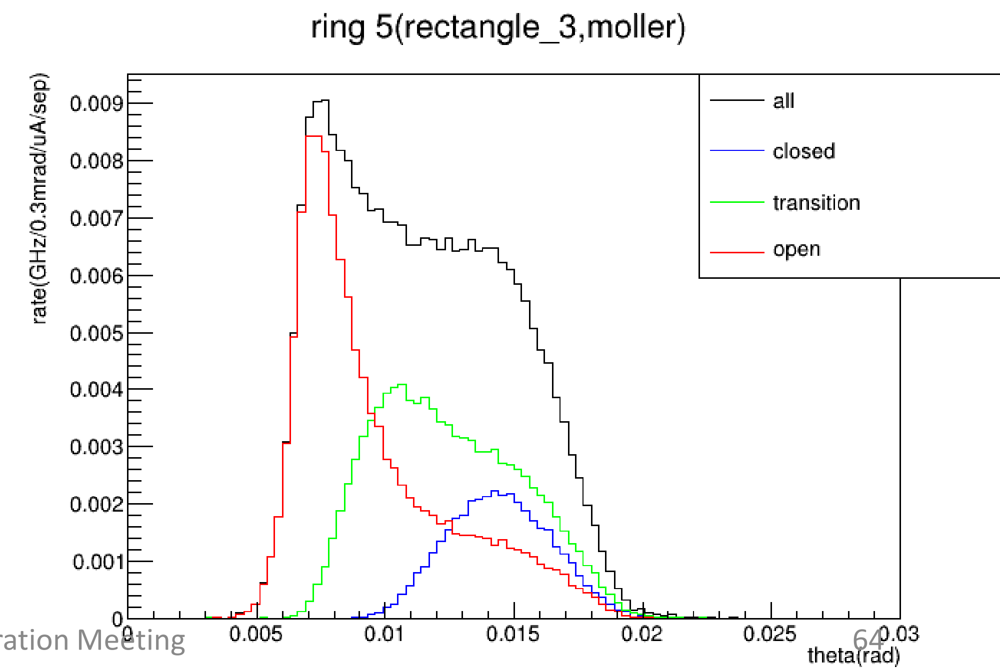
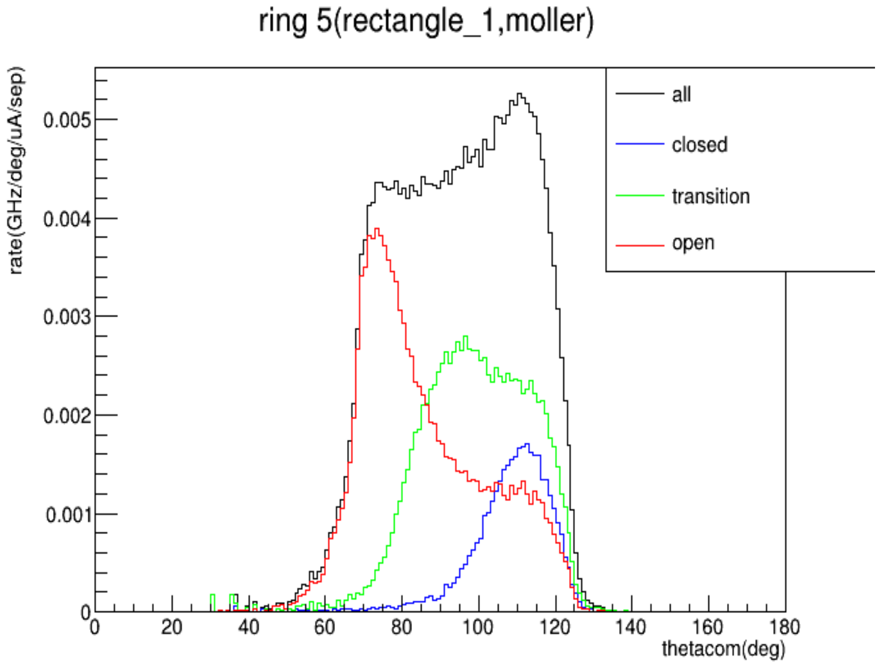
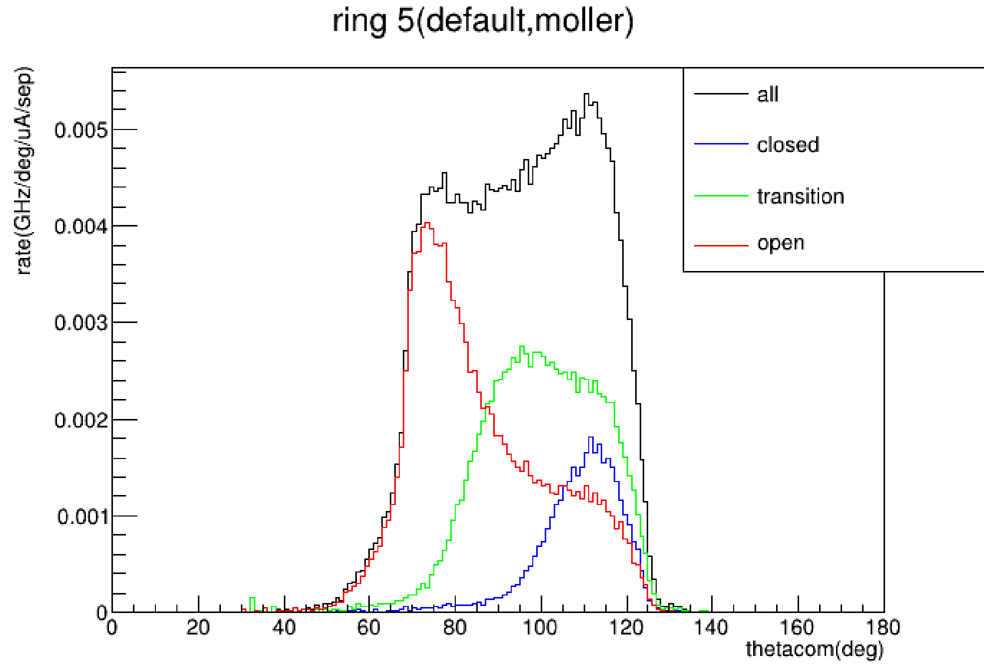
rate(GHz/0.3mrad/uA/sep)
theta(rad)
all
closed
transition
open


ring 5(rectangle_3,moller)
rate(GHz/0.3mrad/uA/sep)
theta(rad)
all
closed
transition
open


ring 5(rectangle_1,moller)
rate(GHz/deg/uA/sep)
thetacom(deg)
all
closed
transition
open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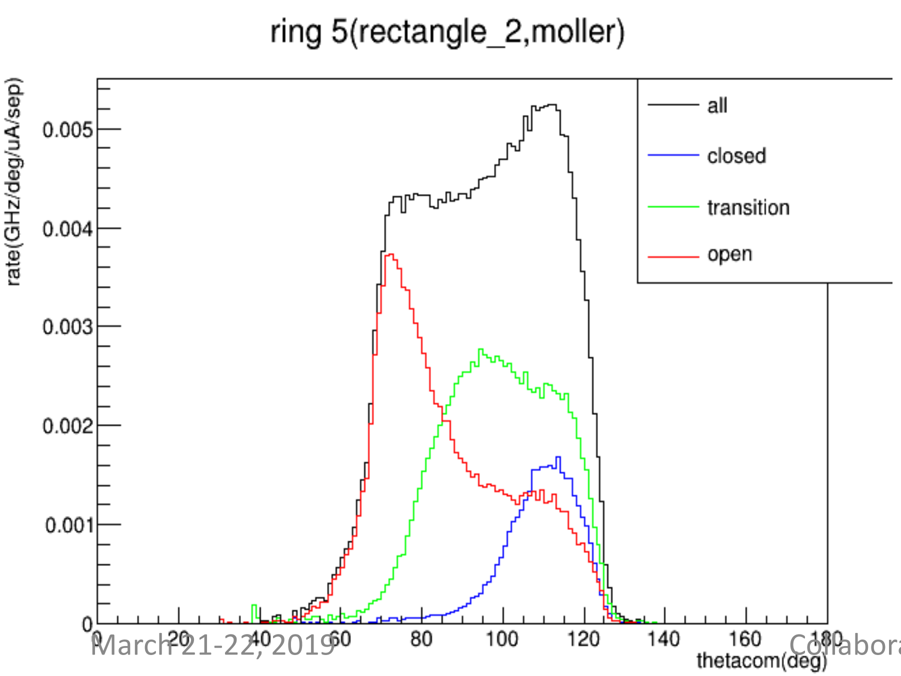
rate(GHz/deg/uA/sep)
thetacom(deg)
all
closed
transition
open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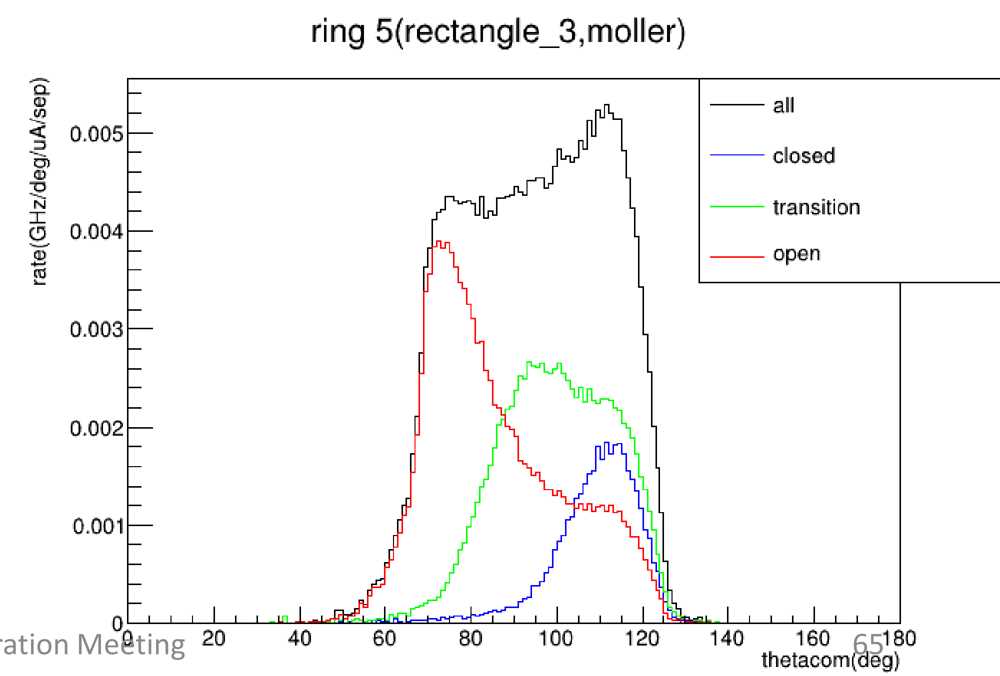
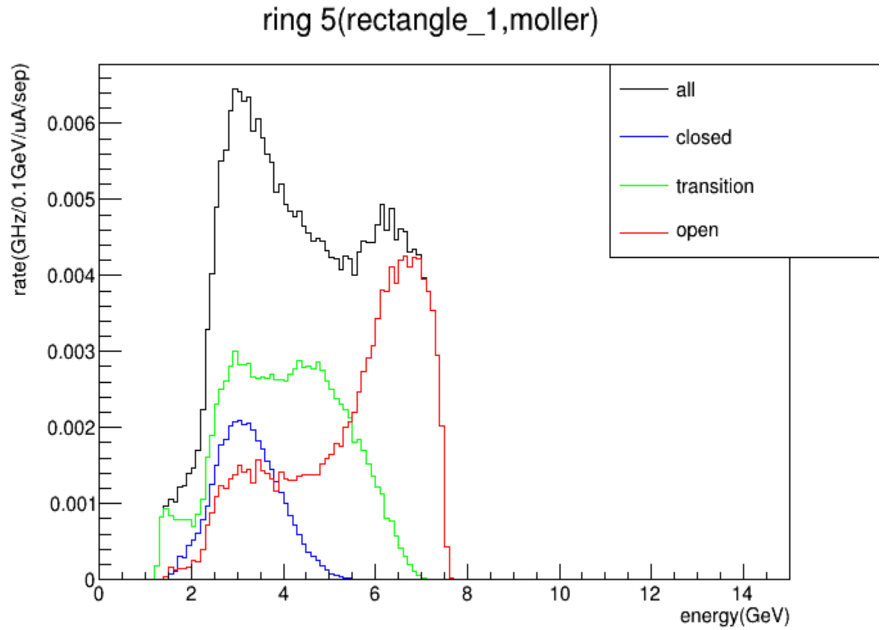
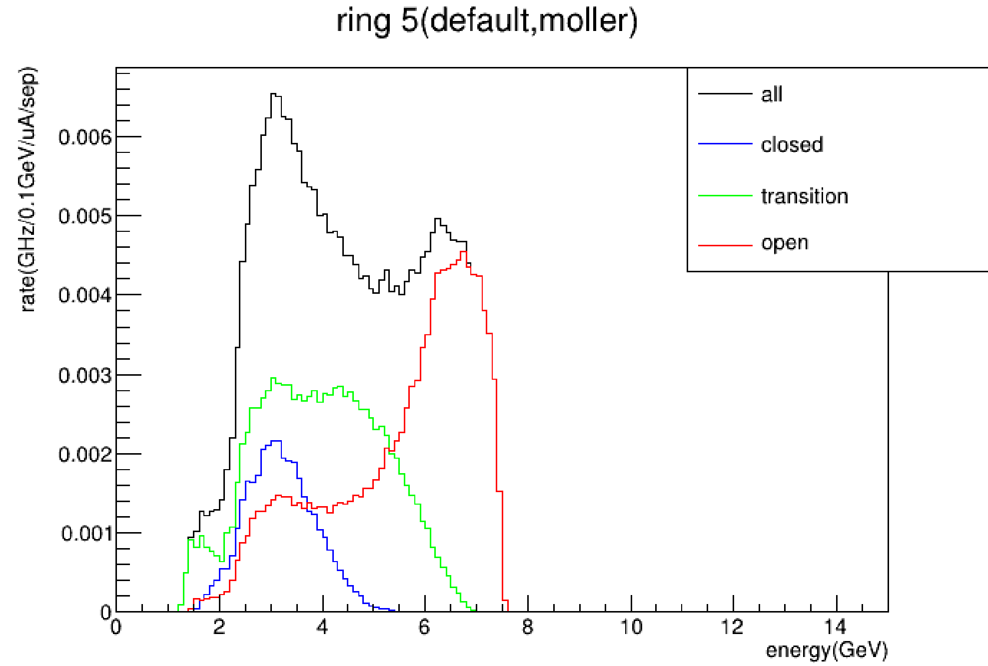
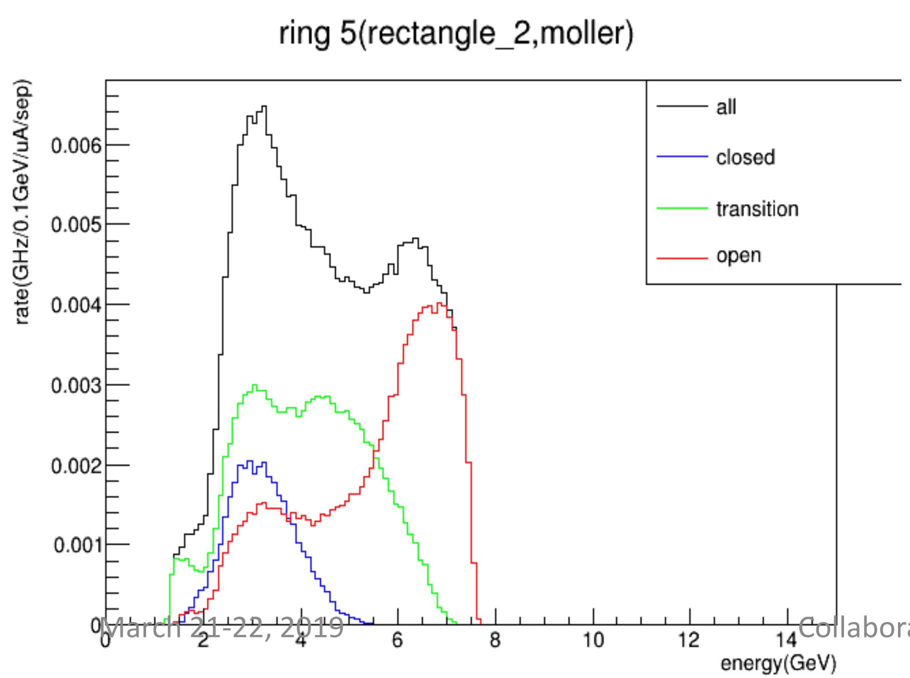
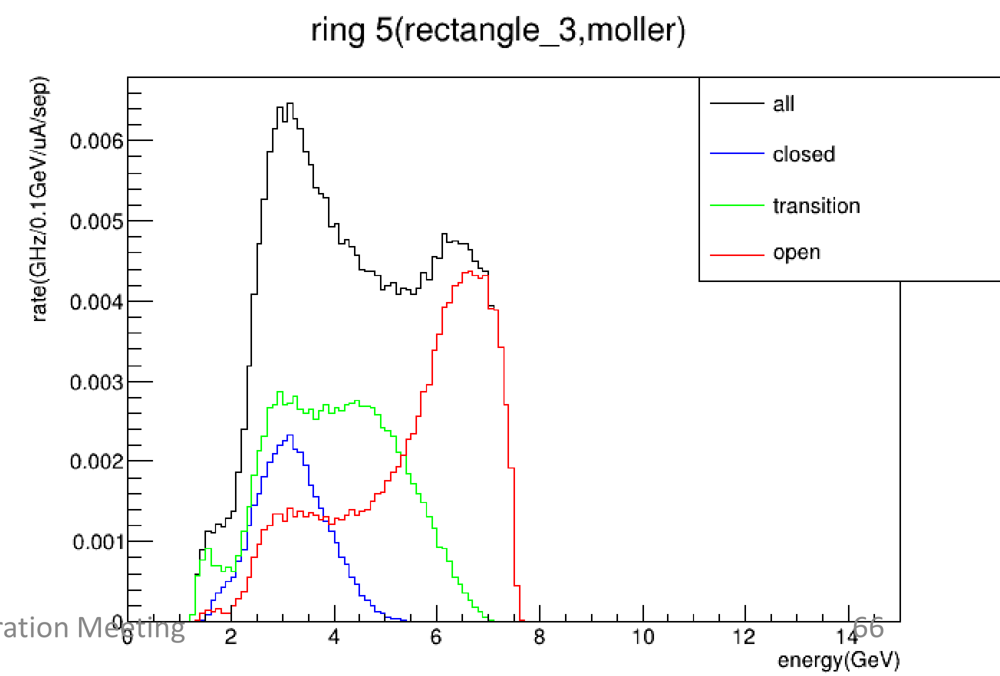
rate(GHz/deg/uA/sep)
thetacom(deg)
all
closed
transition
open


ring 5(rectangle_3,moller)
rate(GHz/deg/uA/sep)
thetacom(deg)
all
closed
transition
open

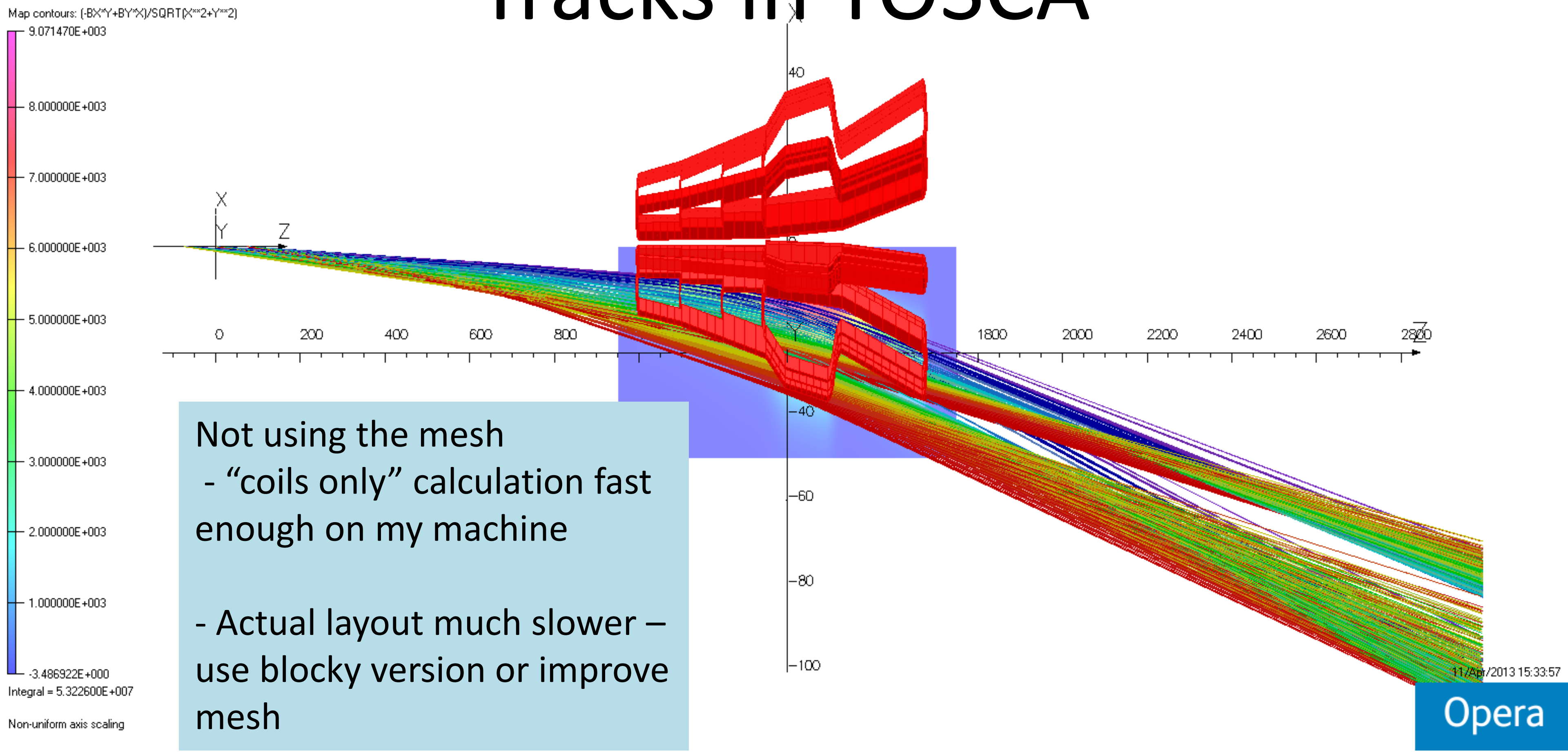

ring 5(rectangle_1,moller)
rate(GHz/0.1GeV/uA/sep)
energy(GeV)
all
closed
transition
open


ring 5(default,moller)
rate(GHz/0.1GeV/uA/sep)
energy(GeV)
all
closed
transition
open


ring 5(rectangle_2,moller)
rate(GHz/0.1GeV/uA/sep)
energy(GeV)
all
closed
transition
open


ring 5(rectangle_3,moller)
rate(GHz/0.1GeV/uA/sep)
energy(GeV)
all
closed
transition
open

Tracks in TOSCA



Not using the mesh
- "coils only" calculation fast enough on my machine

- Actual layout much slower – use blocky version or improve mesh

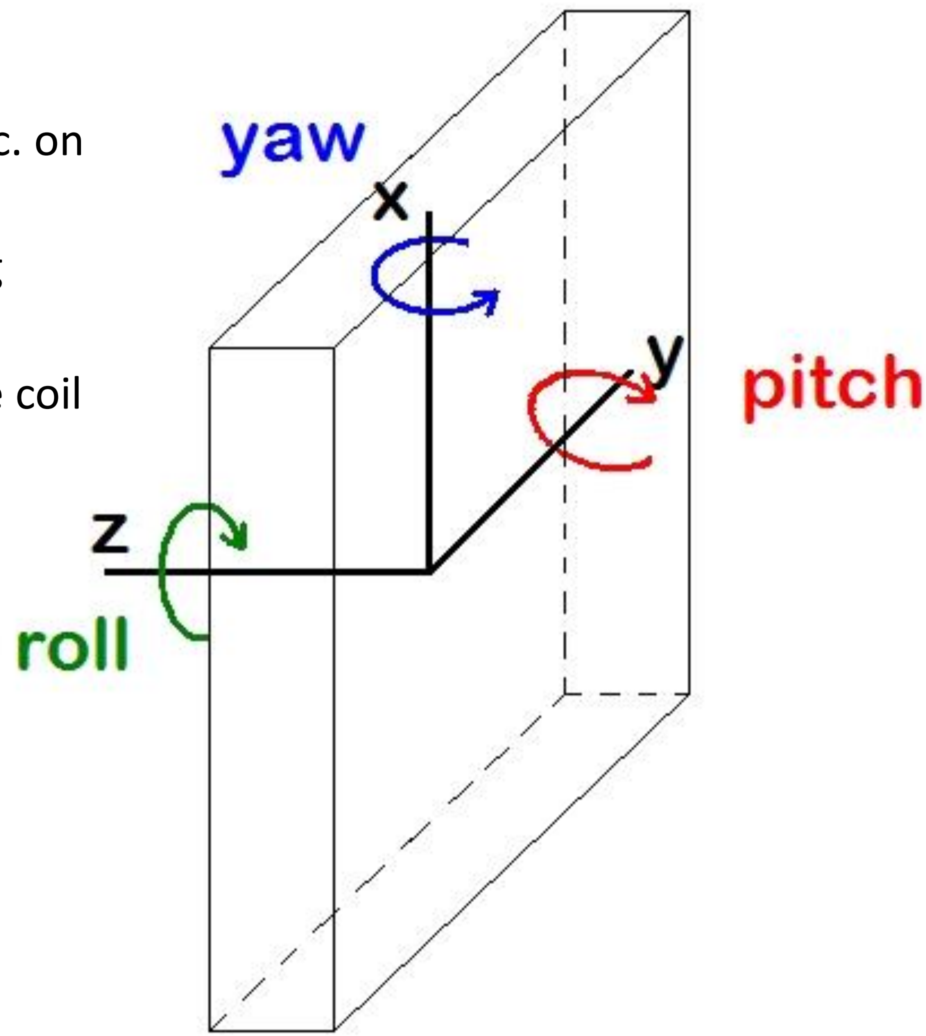
up (z0 =-75 cm) 5.5 to 15 mrads
middle (z0 =0 cm) 6.0 to 17 mrads
down (z0 =75 cm) 6.5 to 19 mrads

All phi values

Tracks colored by theta from purple to red (low to high)

Sensitivity Studies

- Need to consider the effects of asymmetric coils, misalignments etc. on acceptance
- This could affect our manufacturing tolerances and support structure
- Have created field maps for a single coil misplaced by five steps in:
  - -1° < pitch < 1°
  - -4° < roll < 4°
  - -1° < yaw < 1°
  - -2 < r < 2 cm
  - -10 < z < 10 cm
  - -5° < ϕ < 5°
- Simulations need to be run and analyzed



Assuming $Q^2 = 4EE'sin^2\frac{\theta_{lab}}{2}$

The uncertainty on Q^2 is:

$$\delta Q^2 = \left(\frac{\partial Q^2}{\partial E}\right)^2 (\delta E)^2 + \left(\frac{\partial Q^2}{\partial E'}\right)^2 (\delta E')^2 + \left(\frac{\partial Q^2}{\partial \theta_{lab}}\right)^2 (\delta \theta_{lab})^2$$

$\frac{\partial Q^2}{\partial E} = 4E'sin^2\frac{\theta_{lab}}{2}$ ~ 0.001 GeV

$\frac{\partial Q^2}{\partial E'} = 4Esin^2\frac{\theta_{lab}}{2}$ ~ 0.001 GeV

$\frac{\partial Q^2}{\partial \theta_{lab}} = 4EE'sin\frac{\theta_{lab}}{2}cos\frac{\theta_{lab}}{2}$ ~ 1.33 GeV^2/rad

$$\frac{\delta Q^2}{1.33GeV^2/rad} = \delta\theta_{lab} = \frac{(0.005)(.0058\text{GeV}^2)}{1.33GeV^2/rad} = 2\times 10^{-5}rad$$

Slopes give, for example, $\frac{\partial A_{raw}}{\partial z}$

Then $\delta A_{raw} \left(\frac{\partial A_{raw}}{\partial z}\right)^{-1} = \delta z$, the uncertainty in z allowed

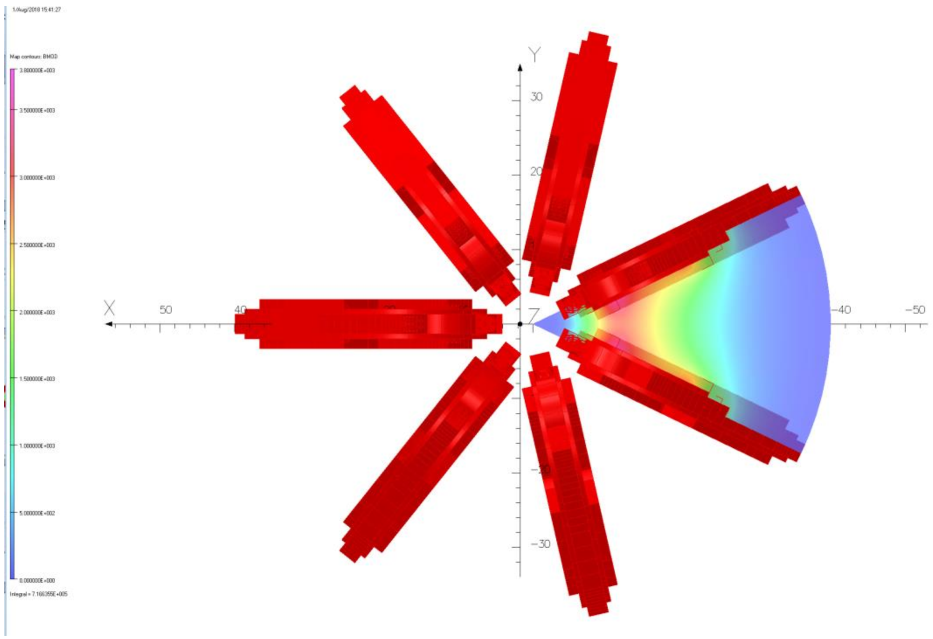
What are the relevant δR, δA, $\delta\theta_{lab}$, $\delta\theta_{com}$?

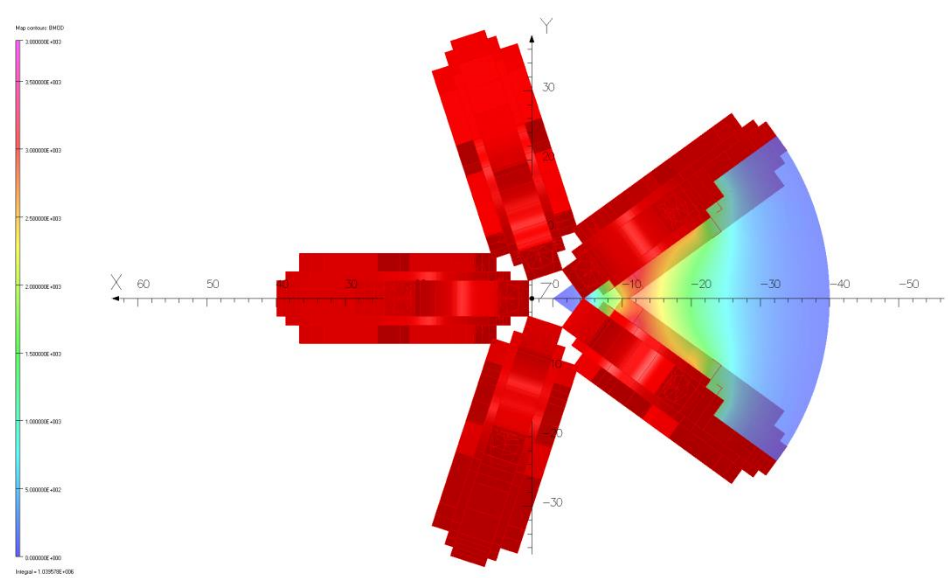
We'll measure a certain rate R and asymmetry A in each septant.
We assume the allowable uncertainty on A to be 0.1 ppb

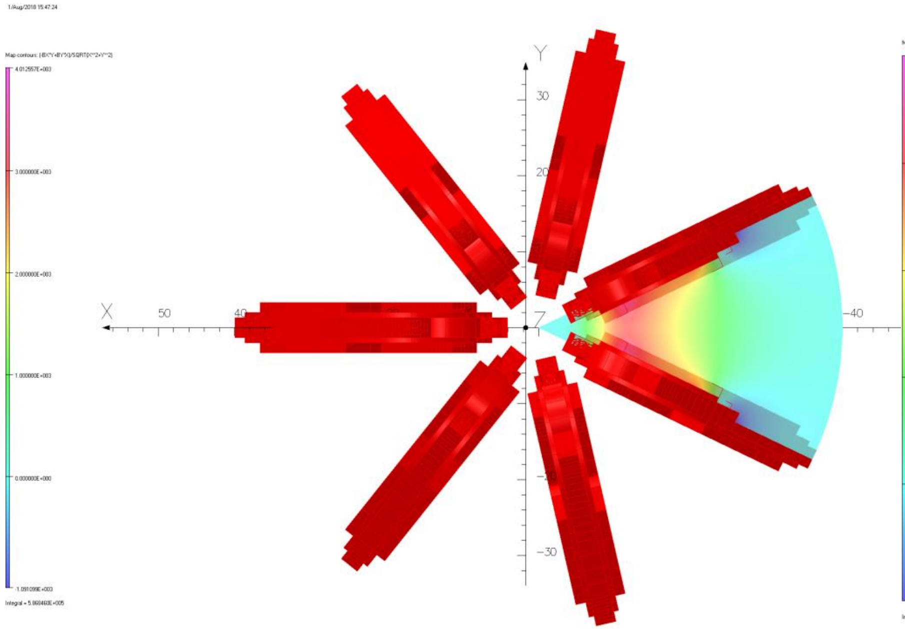
Our ability to determine θ_{lab} in that septant may also be important.

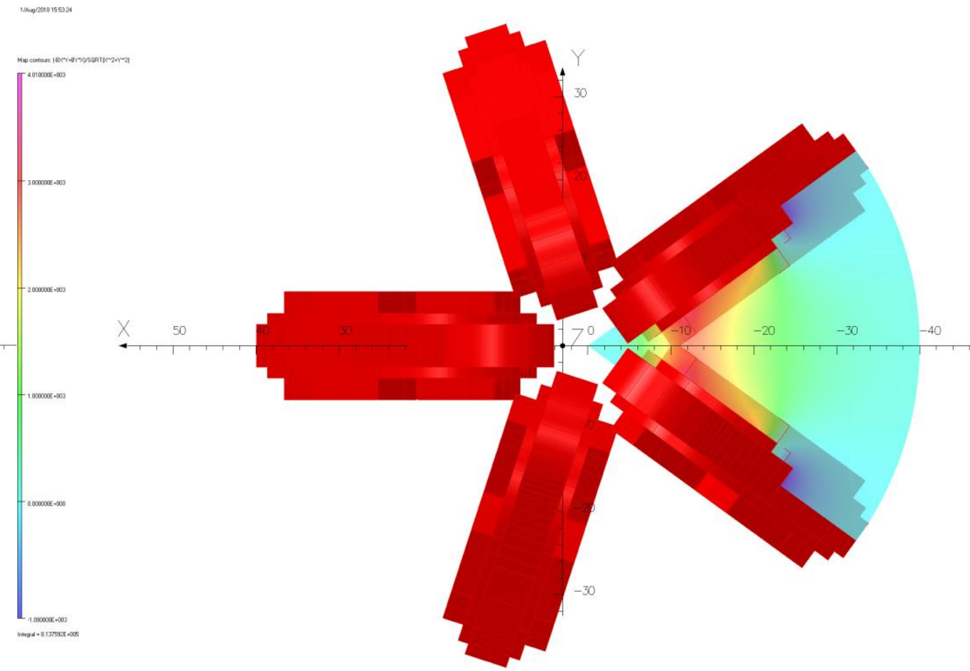
5-fold vs. 7-fold symmetry

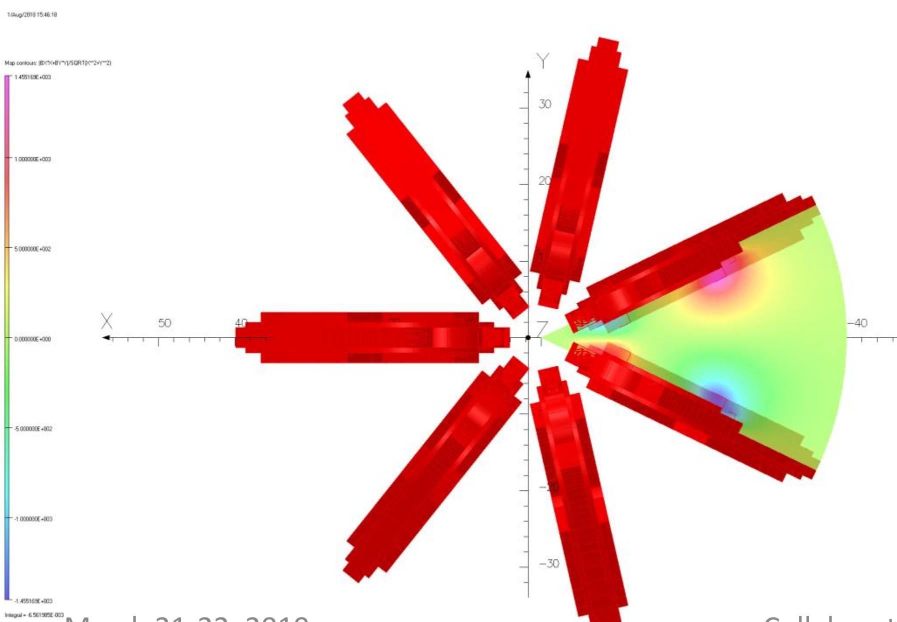
- Adjusted width of the azimuthal part of the conductor cross section
- Factor used was (7/5)^2 (want it all to stay within same radius)
- Some overlaps so not really possible, but close to conductor layout
- Current density adjusted so that there is all the same current along z

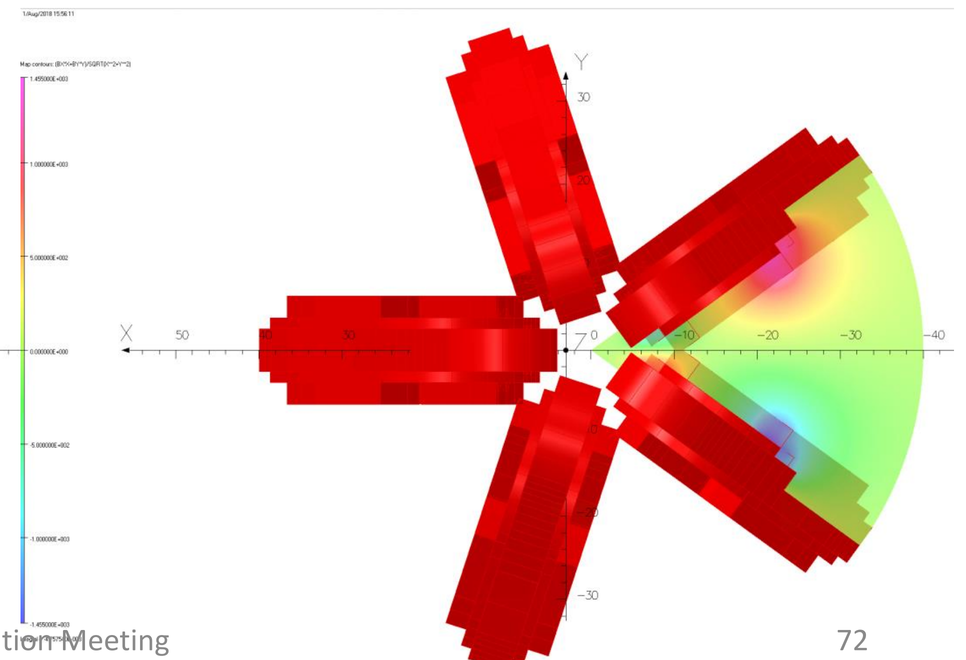




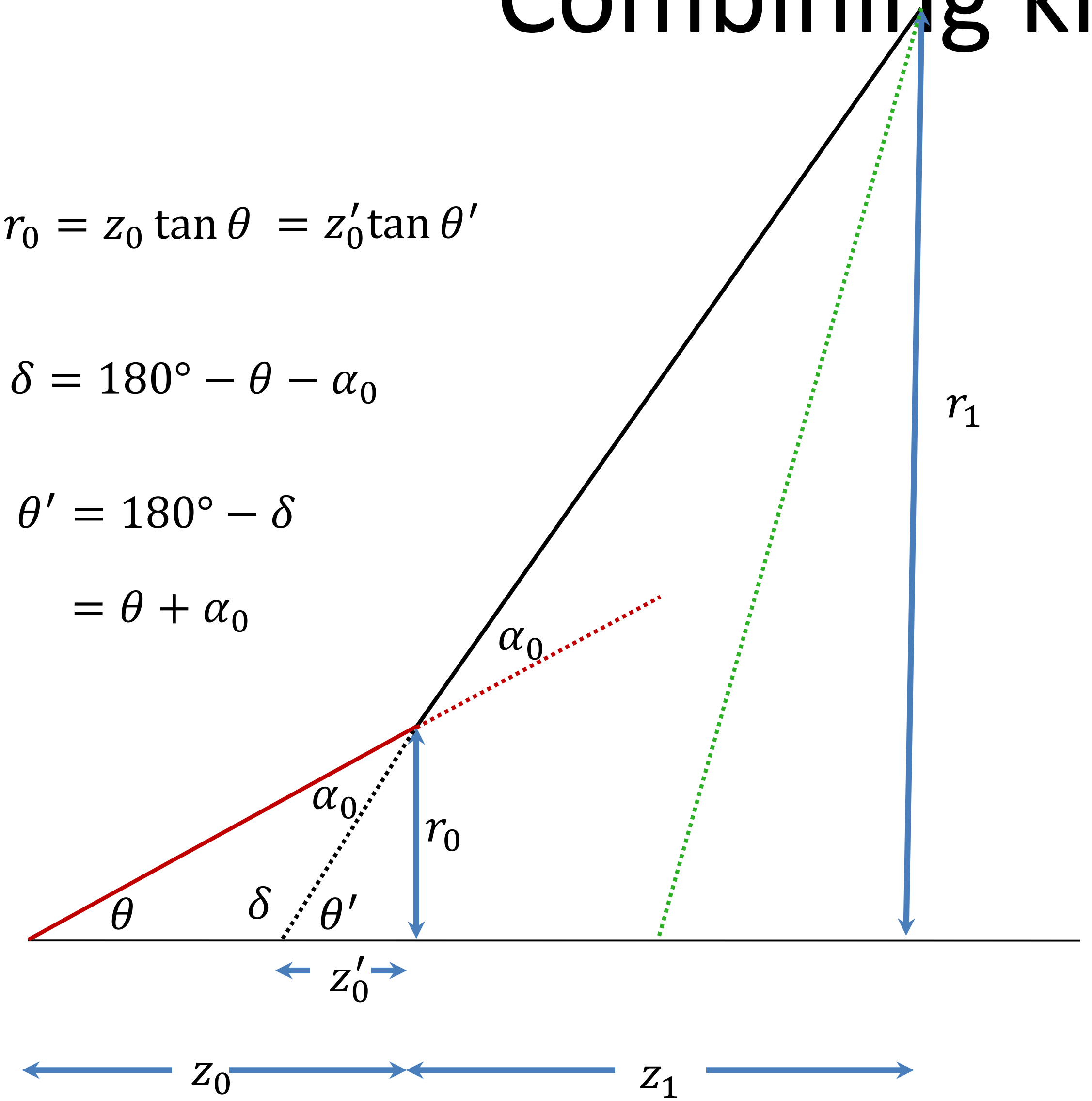

Y
X
50
30
-30
-40


Y
X
50
30
-30
0
-10
-20
-40


Y
X
50
30
-30
-40


Y
X
50
30
-30
0
-10
-20
-40

Combining kicks



$$r_0 = z_0 \tan\theta = z_0' \tan\theta'$$

$$\delta = 180° - \theta - \alpha_0$$

$$\theta' = 180° - \delta$$

$$= \theta + \alpha_0$$

$$r_1 = (z_0' + z_1)\tan\theta'$$

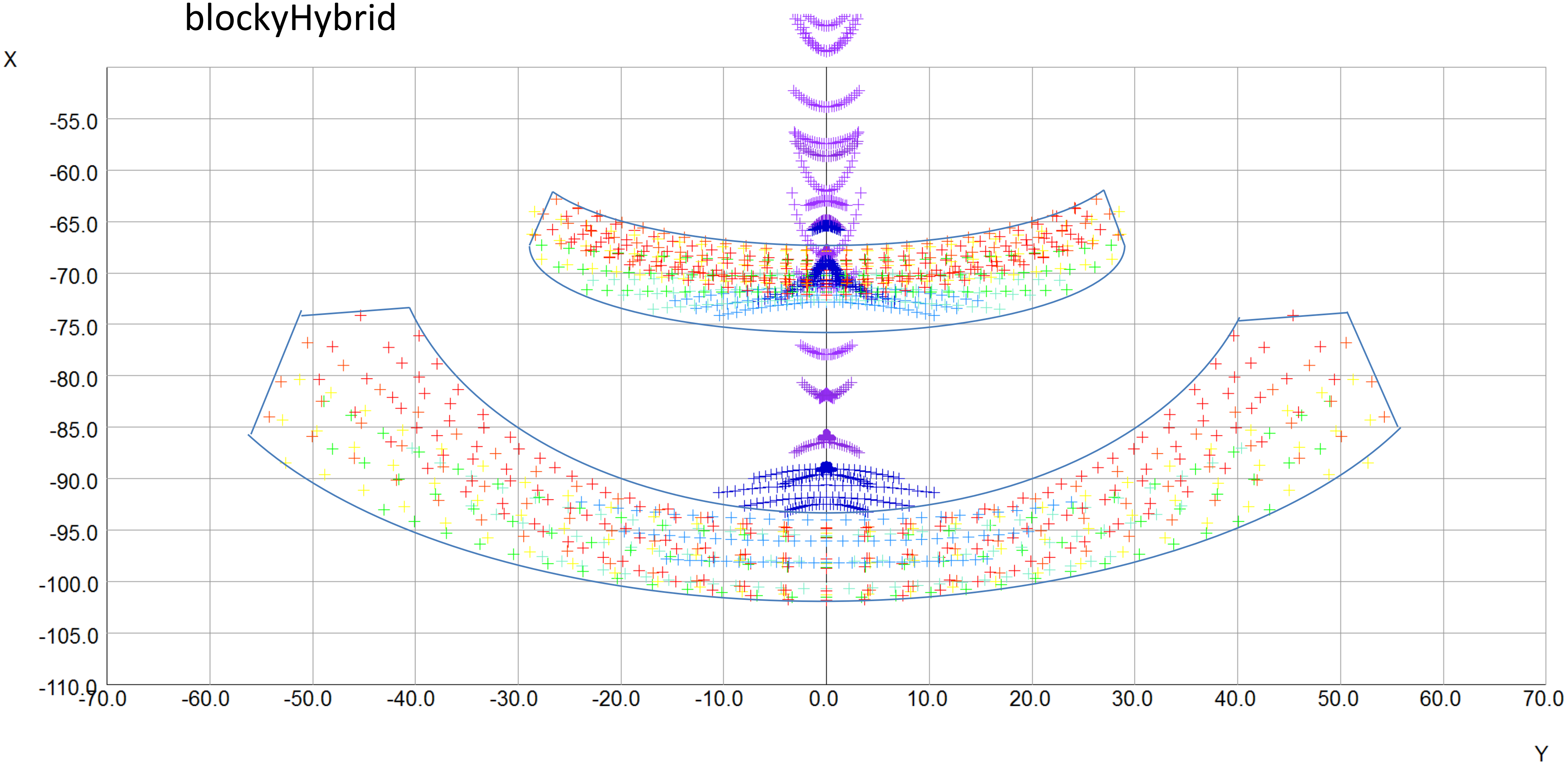
$$= \left(\frac{r_0}{\tan\theta'} + z_1\right)\tan(\theta')$$

$$= r_0 + z_1\tan(\theta')$$

$$= r_0 + z_1\tan(\theta + \alpha_0)$$

$$r_i = r_{i-1} + z_i \tan\left(\theta + \sum_{j=0}^{i-1} \alpha_j\right)$$

blockyHybrid



blockyHybrid

5-fold instead of 7-fold symmetry