Stony Brook University

Systematic uncertainties associated with backgrounds

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Moller Collaboration meeting

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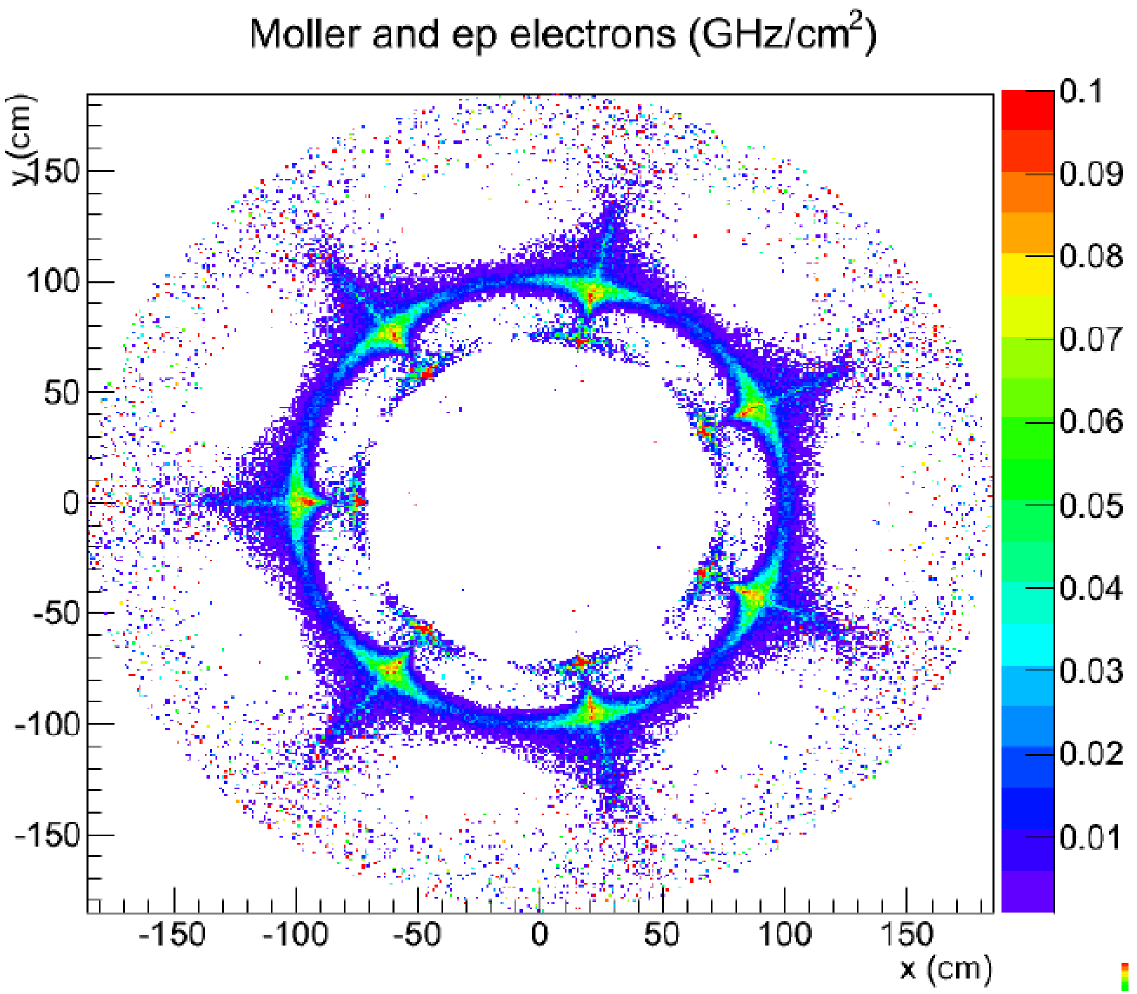
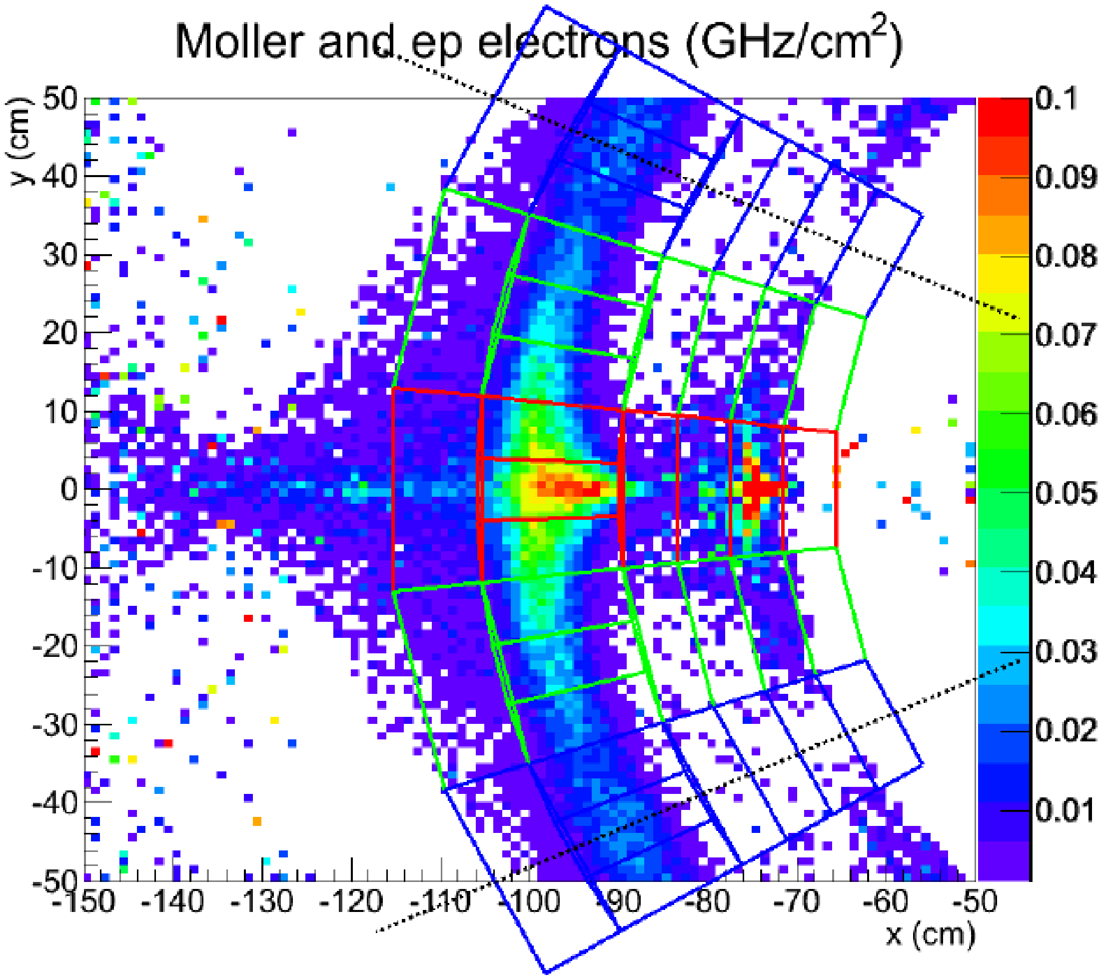
Outline

- Introduction of how I calculate the systematic uncertainties
- Rate and asymmetry estimations, systematic uncertainty tables
- Summary and discussions

Hits on detector plane





18 independent measurements effectively

Introduction of “overall” analysis

- In each quartz tile:

$$A^{(r,\phi)}_{measured} = \frac{N_{ee}}{N_{total}} A^{(r,\phi)}_{ee} + \frac{N_{ep-elastic}}{N_{total}} A^{(r,\phi)}_{ep-elastic} + \frac{N_{ep-inelastic}}{N_{total}} A^{(r,\phi)}_{ep-inelastic} + \frac{N_{eAl-elastic}}{N_{total}} A^{(r,\phi)}_{eAl-elastic} + \frac{N_{eAl-inelastic}}{N_{total}} A^{(r,\phi)}_{eAl-inelastic} \quad \cdots$$

Introduction of "overall" analysis

- In each quartz tile:

$$A^{(r,\phi)}_{measured} = \frac{N_{ee}}{N_{total}} A^{(r,\phi)}_{ee} + \frac{N_{ep-elastic}}{N_{total}} A^{(r,\phi)}_{ep-elastic} + \frac{N_{ep-inelastic}}{N_{total}} A^{(r,\phi)}_{ep-inelastic} + \frac{N_{eAl-elastic}}{N_{total}} A^{(r,\phi)}_{eAl-elastic} + \frac{N_{eAl-inelastic}}{N_{total}} A^{(r,\phi)}_{eAl-inelastic} \dots$$

- 18 tiles segmented in (r, ϕ) space allow us to extract 18 physical asymmetries at most in principle, assuming we know how asymmetry evolves

Introduction of "overall" analysis

- In each quartz tile:

$$A^{(r,\phi)}_{measured} = \frac{N_{ee}}{N_{total}} A^{(r,\phi)}_{ee} + \frac{N_{ep-elastic}}{N_{total}} A^{(r,\phi)}_{ep-elastic} + \frac{N_{ep-inelastic}}{N_{total}} A^{(r,\phi)}_{ep-inelastic} + \frac{N_{eAl-elastic}}{N_{total}} A^{(r,\phi)}_{eAl-elastic} + \frac{N_{eAl-inelastic}}{N_{total}} A^{(r,\phi)}_{eAl-inelastic} \cdots$$

- 18 tiles segmented in (r, ϕ) space allow us to extract 18 physical asymmetries at most in principle, assuming we know how asymmetry evolves

$$\chi^2 = \sum \frac{(A^i_m - f^i_{ee} A^i_{ee} - f^i_{ep-elastic} A^i_{ep-elastic} - f^i_{ep-inelastic} A^i_{ep-inelastic} - f^i_{eAl-elastic} A^i_{eAl-elastic} - f^i_{eAl-inelastic} A^i_{eAl-inelastic})^2}{\sigma^2_{A^i_m}}$$

Introduction of “overall” analysis

$$\chi^2 = \sum \frac{(A^i_m - f^i_{ee}A^i_{ee} - f^i_{ep-elastic}A^i_{ep-elastic} - f^i_{ep-inelastic}A^i_{ep-inelastic} - f^i_{eAl-elastic}A^i_{eAl-elastic} - f^i_{eAl-inelastic}A^i_{eAl-inelastic})^2}{\sigma^2_{A^i_m}}$$

$$\frac{1}{2}\frac{\partial\chi^2}{\partial^2 A^i_{ee}} = \sum_i \frac{(f^i_{ee})^2}{\sigma^2_{A^i_m}}$$

Therefore, the inversed error matrix F is [3]

$$F = \begin{pmatrix} \sum_i \frac{(f^i_{ee})^2}{\sigma^2_{A^i_m}} & \sum_i \frac{f^i_{ee}f^i_{ep-elastic}}{\sigma^2_{A^i_m}} & \cdots \\ \sum_i \frac{f^i_{ee}f^i_{ep-elastic}}{\sigma^2_{A^i_m}} & \sum_i \frac{(f^i_{ep-elastic})^2}{\sigma^2_{A^i_m}} & \cdots \\ \cdots & & \end{pmatrix}$$

F^{-1} is the error matrix for all the physical asymmetries including correlations

Estimating systematic uncertainties

- "overall analysis" takes advantage of all the data collected on our detector plane

- The extracted physical asymmetries can be used to estimate systematic uncertainties due to contaminations in each quartz tile

$$A_{ee} = \frac{N_{tot}}{N_{ee}} A_m - \frac{N_{ep\ elastic}}{N_{ee}} A_{ep\ elastic} - \frac{N_{ep\ inelastic}}{N_{ee}} A_{ep\ inelastic} - \ldots$$

Systematic uncertainty due to contamination is:

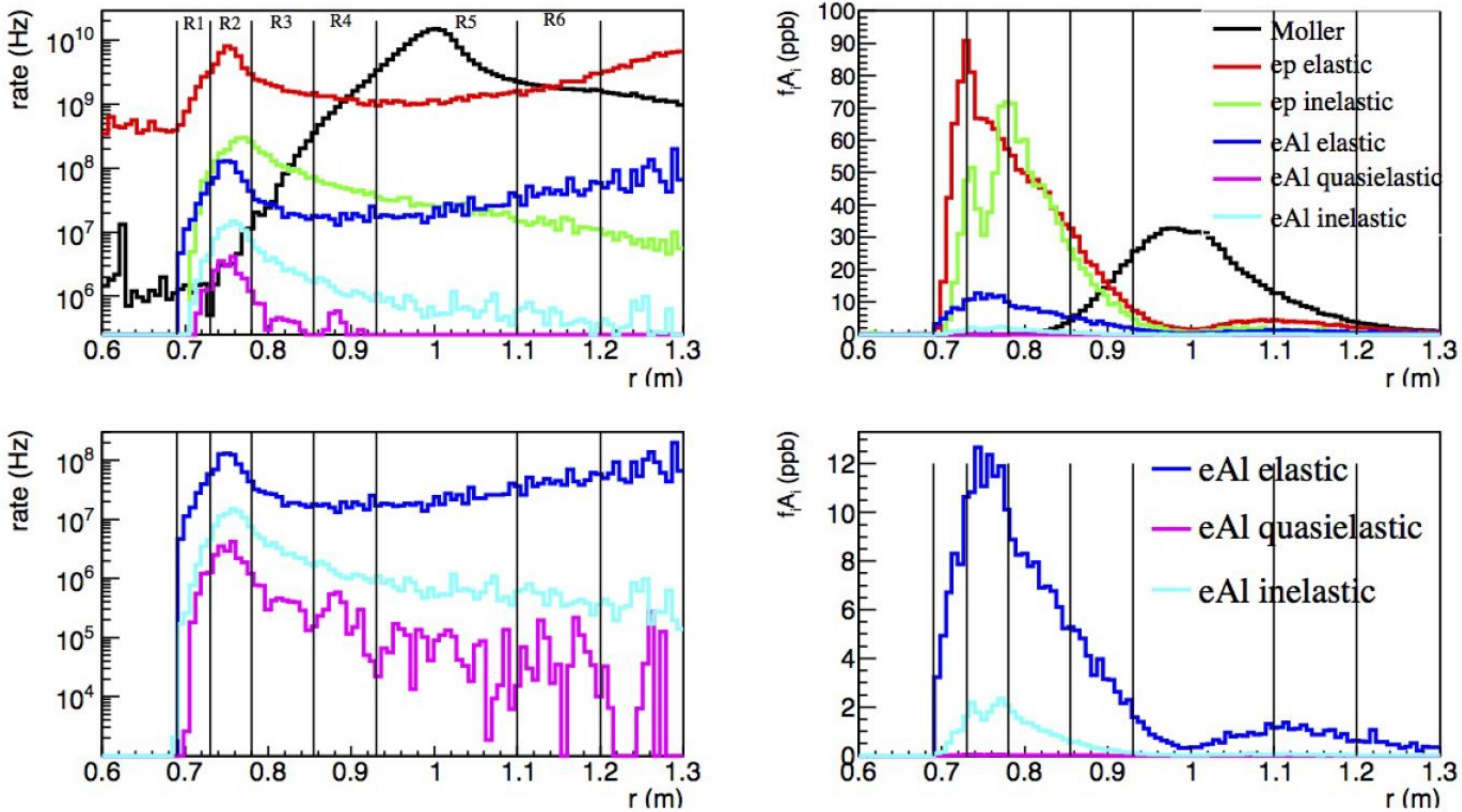
$$\frac{N_{ep\ elastic}}{N_{ee}} \sigma_{A_{ep\ elastic}}, \quad \frac{N_{ep\ inelastic}}{N_{ee}} \sigma_{A_{ep\ inelastic}} \quad \ldots$$

Numbers used in my study

- Beam time: 235 + 95 + 14 = 344 days
- Beam polarization: 80%
- Beam current: 85 muA
- Asymmetries are calculated in terms of R-L asymmetries

✓ To scale to 90% beam polarization and 60 muA, you have to multiply a factor of 1.058 to the projected errors in this slides

Rates and asymmetries



Asymmetry contributions in all 18 quartz tiles

Very small contributions
Will be measured

ring number	sector ID	$\frac{\sigma_A}{A}$ (%)	A_m (ppb)	$\frac{f_{moller}A_{moller}}{A_m}$ (%)	$\frac{f_{epelastic}A_{epelastic}}{A_m}$ (%)	$\frac{f_{epinelastic}A_{epinelastic}}{A_m}$ (%)	$\frac{f_{eAlelastic}A_{eAlelastic}}{A_m}$ (%)	$\frac{f_{eAlquasielastic}A_{eAlquasielastic}}{A_m}$ (%)	$\frac{f_{eAlinelastic}A_{eAlinelastic}}{A_m}$ (%)	$\frac{f_{pim}A_{pim}}{A_m}$ (%)
1	0	9.92	-282.1	0.16	81.6	19.1	-1.06	0	0.23	0
1	1	2.55	-278.2	0	71.7	30.7	-3.29	0	0.87	0
1	2	5.04	-50.92	0	85.7	27.5	-14.6	0	1.34	0
2	0	2.30	-412.2	0	42.5	58.1	-2.50	0	1.77	0.10
2	1	1.18	-203.8	0	56.7	48.0	-6.57	0	1.79	0.06
2	2	1.88	-68.51	0	75.2	39.2	-16.3	0	1.87	0
3	0	3.54	-288.7	0.07	38.0	61.3	-3.55	0	1.21	2.94
3	1	1.83	-176.0	0.06	45.1	58.5	-6.39	0	1.41	1.28
3	2	2.94	-60.01	2.19	56.7	49.6	-10.5	0	1.29	0.75
4	0	6.72	-134.4	3.38	41.8	44.3	-5.22	0	0.76	14.9
4	1	3.83	-98.73	4.91	48.3	41.2	-8.02	0	0.82	12.7
4	2	4.40	-33.88	49.7	32.6	20.7	-7.24	0	0.52	3.60
5	0	4.74	-31.86	87.3	5.46	3.77	-0.82	0	0.08	4.24
5	1	2.57	-35.00	88.0	5.95	3.50	-1.17	0	0.08	3.60
5	2	2.29	-34.34	88.6	7.42	3.56	-1.87	0	0.10	2.17
6	0	21.9	-25.09	42.9	11.7	11.3	-2.43	0	0.30	36.2
6	1	11.2	-16.92	53.8	22.4	7.60	-5.97	0	0.27	21.9
6	2	9.91	-11.47	60.8	28.1	8.55	-8.73	0	0.32	10.9

Table 1: Asymmetry contributions from all processes for all the 18 measurements. Sector ID, 0=closed, 1=transition, 2=open. $\frac{\sigma_A}{A}$ is negative due to a negative measured asymmetry A_m. "0" indicates the ratio is less than 0.05%.

Asymmetries in 6 rings

ring number	$\frac{\sigma_A}{A}$ (%)	A_m (ppb)	$\frac{f_{moller}A_{moller}}{A_m}$ (%)	$\frac{f_{epelastic}A_{epelastic}}{A_m}$ (%)	$\frac{f_{epinelastic}A_{epinelastic}}{A_m}$ (%)	$\frac{f_{eAlelastic}A_{eAlelastic}}{A_m}$ (%)	$\frac{f_{eAlquasielastic}A_{eAlquasielastic}}{A_m}$ (%)	$\frac{f_{eAlinelastic}A_{eAlinelastic}}{A_m}$ (%)	$\frac{f_{pim}A_{pim}}{A_m}$ (%)
1	3.05	-78.69	0	79.9	28.6	-9.66	0	1.12	0
2	1.09	-103.1	0	65.3	44.1	-11.3	0	1.83	0.05
3	1.68	-91.15	1.12	50.3	54.3	-8.25	0	1.34	1.13
4	3.06	-44.73	33.5	37.8	28.3	-7.33	0	0.63	7.04
5	1.61	-34.26	88.2	6.61	3.56	-1.47	0	0.09	2.98
6	7.24	-13.28	57.5	25.3	8.40	-7.47	0	0.30	15.9

Table 2: Asymmetry contributions from all processes for 6 rings. "0" indicates the ratio is less than 0.05%.

Extracting 4 asymmetries

Overall analysis (ring 5, open sector):

Processes	Expected A (ppb)	Delta_A (ppb)	Delta_A / A
Moller	-35.196	0.628(0.09)	1.78%(0.26%)
Ep-elastic	-19.67	0.959 (0.1)	4.9%(0.5%)
Ep-inelastic	-415.435	16.515(7.3)	4.0%(1.8%)
eAl-elastic	297.3	63.45(20.3)	21.3%(6.8%)

Sys. Error due to subtracting eAl quasielastic, eAl inelastic and pim DIS before overall fitting (assuming 100% uncertainty on eAl asymmetry, 10% on pim)

Sys. Uncertainties for ring 5, open sector

Could be reduced by additional inputs from Qweak measurement

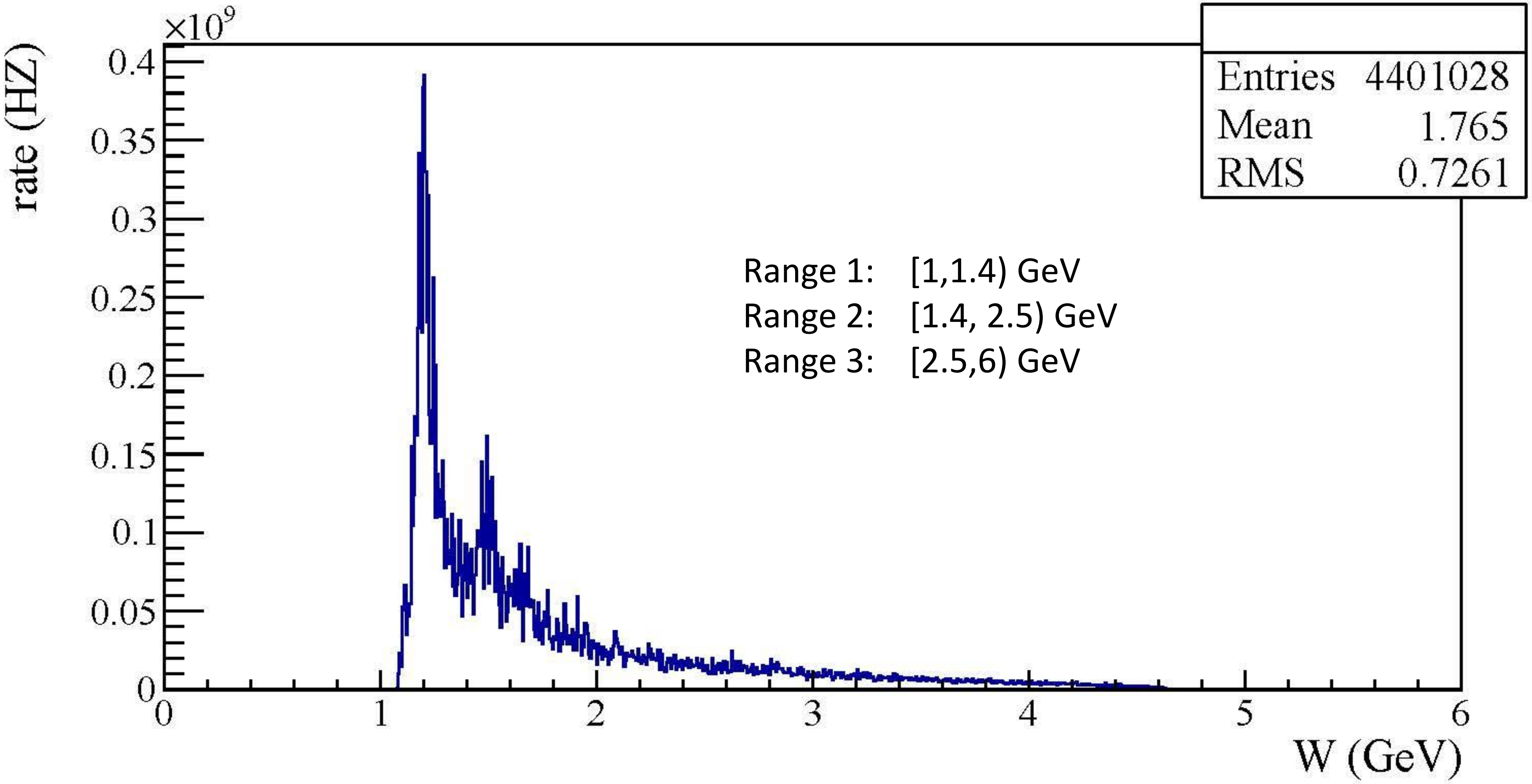
Processes	Error (ppb)	Normalized to 33 ppb
Moller	0.91 (stat.)	2.76%
ep-elastic	0.14(sys.)	0.43% (0.04%)
ep-inelastic	0.056 (sys.)	0.17% (0.08%)
eAl-elastic	0.16 (sys.)	0.48%(0.15%)
eAl-quasielastic	~0	~0
eAl-inelastic	0.038(sys.)	0.1%
Pim	0.086(sys.)	0.26%

Additional sys. Error due to subtracting eAl quasielastic, eAl inelastic and pim DIS before overall fitting (assuming 100% uncertainty on eAl, 10% on pim)

Sys. Uncertainties for all sectors in ring 5

Processes	Error (ppb) Open	Normalized to 33 ppb (%)	Error (ppb) Transition	Normalized to 33 ppb (%)	Error (ppb) Closed	Normalized to 33 ppb (%)	Combined (%)
Moller	0.91 (stat.)	2.76	0.989(stat.)	3	1.65(stat.)	5	1.88
ep-elastic	0.14(sys.)	0.43 (0.04)	0.11(sys.)	0.33(0.035)	0.093(sys.)	0.28(0.029)	0.37(0.036)*
ep-inelastic	0.056 (sys.)	0.17 (0.08)	0.054(sys.)	0.16(0.07)	0.052(sys.)	0.16(0.07)	0.16(0.07)
eAl-elastic	0.16 (sys.)	0.48(0.15)	0.096(sys.)	0.29(0.09)	0.061(sys.)	0.19(0.06)	0.36(0.1)*
eAl-quasi	~0(sys.)	~0	~0(sys.)	~0	~0(sys.)	~0	~0
eAl-inela	0.038(sys.)	0.1	0.031(sys.)	0.09	0.027	0.08	0.09
pim	0.086(sys.)	0.26	0.138(sys.)	0.4	0.148	0.4	0.33

Decompose ep-inelastic into 3 parts



Extracting 6 asymmetries

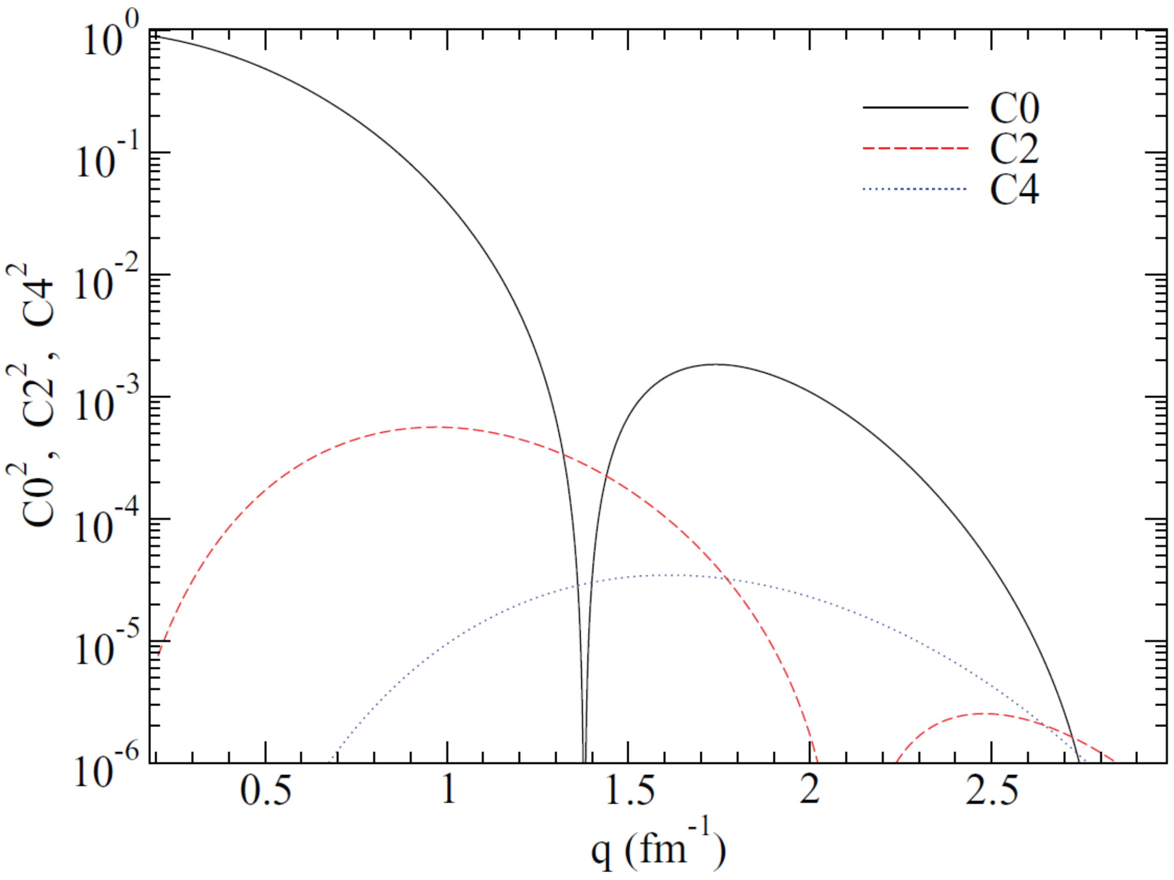
Overall analysis (ring 5, open sector):

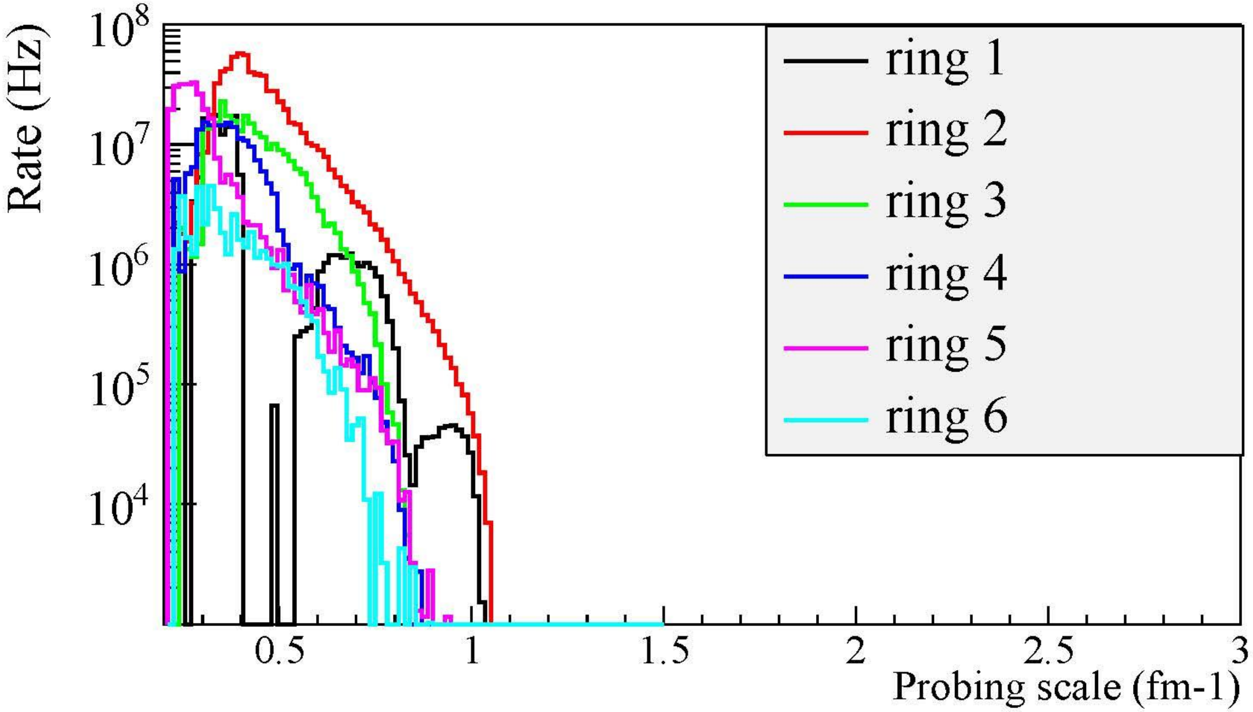
Processes	Expected A (ppb)	Delta_A (ppb)	Delta_A / |A|
Moller	-35.196	0.64(0.1)	1.82%(0.29%)
ep-elastic	-19.67	1.817(0.32)	9.24%(1.6%)
ep-inelastic (range 1)	-439.94	80.6(28.7)	18.3%(6.5%)
ep-inelastic (range 2)	-433.96	38.3(17.9)	8.8%(4.1%)
ep-inelastic (range 3)	-384.59	91.5(33.5)	23.8%(8.7%)
eAl-elastic	297.27	83.01(21.0)	27.9%(7%)

Sys. Uncertainties for ring 5

Processes	Error (ppb) Open	Normalized to 33 ppb (%)	Error (ppb) Transition	Normalized to 33 ppb (%)	Error (ppb) Closed	Normalized to 33 ppb (%)	Combined (%)
Moller	0.91 (stat.)	2.76	0.989(stat.)	3	1.65(stat.)	5	1.88
ep-elastic	0.27(sys.)	0.8 (0.14)	0.21(sys.)	0.64(0.11)	0.176(sys.)	0.53(0.09)	0.7(0.12) *
ep-inelastic (1)	0.07(sys.)	0.22(0.08)	0.072(sys.)	0.22(0.08)	0.07(sys.)	0.21(0.08)	0.22(0.08)
ep-inelastic(2)	0.04(sys.)	0.13(0.06)	0.041(sys.)	0.12(0.06)	0.043(sys.)	0.13(0.06)	0.13(0.06)
ep-inelastic(3)	0.13 (sys.)	0.38 (0.14)	0.12(sys.)	0.35(0.13)	0.11(sys.)	0.33(0.12)	0.36(0.13)
eAl-elastic	0.21 (sys.)	0.6 (0.16)	0.13(sys.)	0.38(0.1)	0.08(sys.)	0.24(0.06)	0.46(0.12) *
eAl-quasi	0	0	0	0	0	0	0
eAl-inela	0.038(sys.)	0.1	0.031	0.09	0.027	0.08	0.09
pim	0.086(sys.)	0.26	0.139	0.42	0.148	0.45	0.33

Probing scale coverage for eAl elastic process





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Summaries for the systematic uncertainties

Extracting 4 asymmetries

Processes	Combined (%)
Moller (stat.)	1.88
ep-elastic(sys)	0.37(0.036)*
ep-inelastic (sys)	0.16(0.07)
eAl-elastic(sys)	0.36(0.1)*
eAl-quasi(sys)	~0
eAl-inela (sys)	0.09
Pim (sys)	0.33

* : can be reduced by additional inputs

Extracting 6 asymmetries

Processes	Combined (%)
Moller (stat.)	1.88
ep-elastic (sys.)	0.7(0.12) *
ep-inelastic (1) (sys.)	0.22(0.08)
ep-inelastic(2) (sys.)	0.13(0.06)
ep-inelastic(3) (sys.)	0.36(0.13)
eAl-elastic (sys.)	0.46(0.12) *
eAl-quasi (sys.)	0
eAl-inela (sys.)	0.09
Pim (sys.)	0.33

Discussions

- For the above study, the pion rates are from Wiser calculations
  - ➢The rate is believed to be higher than the realistic rate
  - ➢Rakitha did some dedicated study on Pion rate (comments from Rakitha???)
  - ➢Need efforts to put in our framework and update the numbers

- The current study doesn’t segment Moller ring into more quartz tiles
  - ➢With more quartz tiles, the correlations between different processes can be reduced

- To combine the individual systematic uncertainties, taking into account correlations of different sources will yield a smaller number given in our write-up
  - ➢In the write-up, we just give a boundary by linear summation of all the numbers

Backups

Asymmetry contributions in all 18 quartz tiles

ring number	sector ID	$\frac{\sigma_A}{A}$ (%)	A_m (ppb)	$\frac{f_{moller}A_{moller}}{A_m}$ (%)	$\frac{f_{epelastic}A_{epelastic}}{A_m}$ (%)	$\frac{f^1_{epinelastic}A^1_{epinelastic}}{A_m}$ (%)	$\frac{f^2_{epinelastic}A^2_{epinelastic}}{A_m}$ (%)	$\frac{f^3_{epinelastic}A^3_{epinelastic}}{A_m}$ (%)	$\frac{f_{eAlelastic}A_{eAlelastic}}{A_m}$ (%)	$\frac{f_{eAlquasielastic}A_{eAlquasielastic}}{A_m}$ (%)	$\frac{f_{eAlinelastic}A_{eAlinelastic}}{A_m}$ (%)	$\frac{f_{pim}A_{pim}}{A_m}$ (%)
1	0	9.92	-282.1	0.16	81.6	19.0	0	0	-1.06	0	0.23	0
1	1	2.55	-278.2	0	71.7	26.7	3.95	0	-3.29	0	0.87	0
1	2	5.04	-50.92	0	85.7	24.4	3.10	0	-14.6	0	1.34	0
2	0	2.30	-412.2	0	42.5	31.6	26.5	0	-2.50	0	1.77	0.1
2	1	1.18	-203.8	0	56.7	27.8	20.1	0	-6.57	0	1.79	0.06
2	2	1.88	-68.51	0	75.2	24.4	14.8	0	-16.3	0	1.87	0
3	0	3.54	-288.7	0.07	38.0	16.6	32.7	12.0	-3.55	0	1.21	2.94
3	1	1.83	-176.0	0.06	45.1	18.5	34.8	5.21	-6.39	0	1.41	1.28
3	2	2.94	-60.01	2.19	56.7	15.0	31.9	2.77	-10.5	0	1.29	0.75
4	0	6.72	-134.4	3.38	41.8	12.8	13.6	18.0	-5.22	0	0.76	14.9
4	1	3.83	-98.73	4.91	48.3	10.8	15.8	14.6	-8.02	0	0.82	12.7
4	2	4.40	-33.88	49.7	32.6	5.48	10.1	5.17	-7.24	0	0.52	3.60
5	0	4.74	-31.86	87.3	5.46	1.09	1.39	1.29	-0.82	0	0.08	4.24
5	1	2.57	-35.00	88.0	5.95	1.02	1.20	1.28	-1.17	0	0.08	3.60
5	2	2.29	-34.34	88.6	7.42	1.01	1.20	1.34	-1.87	0	0.10	2.17
6	0	21.9	-25.09	42.9	11.7	6.92	2.23	2.17	-2.43	0	0.30	36.2
6	1	11.2	-16.92	53.8	22.4	2.32	2.81	2.47	-5.97	0	0.27	21.9
6	2	9.91	-11.47	60.8	28.2	2.41	3.26	2.89	-8.73	0	0.32	10.9

Table 3: Asymmetry contributions from all processes (ep-inelastic into 3 processes) for all the 18 quartz. Sector ID, 0=closed, 1=transition, 2=open. $\frac{\sigma_A}{A}$ is negative due to a negative measured asymmetry A_m. "0" indicates the ratio is less than 0.05%.

Asymmetry contributions in 6 rings

ring number	$\frac{\sigma_A}{A}$ (%)	A_m (ppb)	$\frac{f_{moller}A_{moller}}{A_m}$ (%)	$\frac{f_{epelastic}A_{epelastic}}{A_m}$ (%)	$\frac{f^1_{epinelastic}A^1_{epinelastic}}{A_m}$ (%)	$\frac{f^2_{epinelastic}A^2_{epinelastic}}{A_m}$ (%)	$\frac{f^3_{epinelastic}A^3_{epinelastic}}{A_m}$ (%)	$\frac{f_{eAlelastic}A_{eAlelastic}}{A_m}$ (%)	$\frac{f_{eAlquasielastic}A_{eAlquasielastic}}{A_m}$ (%)	$\frac{f_{eAlinelastic}A_{eAlinelastic}}{A_m}$ (%)	$\frac{f_{pim}A_{pim}}{A_m}$ (%)
1	3.05	-78.69	0	79.9	25.2	3.37	0	-9.66	0	1.12	0
2	1.09	-103.1	0	65.3	26.3	17.7	0	-11.3	0	1.83	0.05
3	1.68	-91.15	1.12	50.3	16.6	33.2	4.48	-8.25	0	1.34	1.13
4	3.06	-44.73	33.5	37.8	7.54	12.0	8.80	-7.33	0	0.63	7.04
5	1.61	-34.26	88.2	6.61	1.03	1.22	1.31	-1.47	0	0.09	2.98
6	7.24	-13.28	57.5	25.3	2.64	3.05	2.71	-7.47	0	0.30	15.9

Table 4: Asymmetry contributions from all processes (ep-inelastic into 3 processes) for 6 rings. “0” indicates the ratio is less than 0.05%.

Combination of data in Moller ring

- Data for different phi sectors are combined by statistical errors of Moller projections:

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}$$

- Systematic errors due to contaminations are considered to be fully correlated, since their estimations are from the same source

$$\mathrm{d}a = \frac{\frac{da_1}{\sigma_1^2}+\frac{da_2}{\sigma_2^2}+\frac{da_3}{\sigma_3^2}}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}}$$

moller stat: 1.88%
ep-elastic sys: 0.37%
ep-inelastic sys: 0.16%
eAl-elastic sys: 0.36%

Extracting Moller and ep-inelastic(3) asymmetries
---assuming well known of ep-elastic and eAl-elastic

Will be a constant while doing differential to chi2 to get error matrix,
Now error matrix becomes 4 x 4

$$\chi^2 = \sum \frac{(A^i_m - f^i_{ee}A^i_{ee} - f^i_{ep-elastic}A^i_{ep-elastic} - f^i_{ep-inelastic}A^i_{ep-inelastic} - f^i_{eAl-elastic}A^i_{eAl-elastic})^2}{\sigma^2_{A^i_m}}$$

-------------------------------- overall ----------------------------------

ee:	-35.19 ppb	0.635 ppb	-1.8%
epinelastic 1 :	-439.9 ppb	28.13 ppb	-6.4%
epinelastic 2 :	-434.0 ppb	33.80 ppb	-7.8%
epinelastic 3 :	-384.6 ppb	78.33 ppb	-20.4%